DEVELOPMENT OF COMPUTER PROGRAMS FOR ANALYSIS OF SINGLE PANEL AND CONTINUOUS RECTANGULAR PLATES

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CERTIFICATION

This is to certify that this Research Work titled **"Development of Computer Programs for Analysis of Single Panel and Continuous Rectangular Plates"** was actually carried out by **ADAH, EDWARD INGIO** with Registration Number **20124766148** of the Department of Civil Engineering, Federal University of Technology, Owerri and that this work has not been submittedto any University before now.

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DEDICATION

This research work is dedicated to my dear wife, Mrs. Anastacia Chinenye, Adah and our children, namely Faith and Joshua, who stood by me and supported me to realize this dream, and to the Holy Spirit, for His inspiration and guidance.

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TABLE OF CONTENT

LIST OF TABLES

LIST OF FIGURES

NOTATIONS

w = Deflection

w_{max} = Maximum Deflection

w^R = Deflection in X- Direction for Non-dimensional Parameters

w^Q = Deflection in Y- Direction for Non- dimensional Parameters

A = Amplitude

R = Non dimensional Parameter in X- direction (=X/a)

Q = Non dimensional Parameter in Y- direction (=Y/b)

$\int$ = Definite Integral from 0 to 1

P = Aspect Ratio (= a/b)

S= Aspect Ratio (=b/a)

a = Dimension along R direction

b = Dimension along Q direction

u_p = Coefficient of amplitude in term of P

u_s or u = Coefficient of amplitude in terms of S

α = Deflection Coefficient

β or beta = Moment Coefficient Considered at the center along R- direction

$\beta_1$or beta1= Moment Coefficient Considered at the center along Q- Direction

β_2 or beta2 = Moment Coefficient considered at the clamped edge along R direction

β_3 or beta3 = Moment Coefficient considered at the clamped edge along Q direction

v = Poisson Ratio

δ or delta = Shear force Coefficient along R direction

δ_1 or delta1= Shear Force Coefficient along Q direction

M_{xmax} = Maximum Moment at the center of plate in X direction

M_{ymax}= Maximum Moment at the center of plate in Y direction

M_{xc} = Moment at the center of plate in X direction

M_{yc}= Moment at the center of plate in Y direction

M_{xe} = Moment at the clamped edge in X direction

M_{ye} = Moment at the clamped edge in Y direction

V_x = Shear Force in X direction

V_y= Shear Force in Y direction

Π_x = Total Potential Energy Functional in X direction

Π = Total Potential Energy Functional

N_x = Buckling Load or Critical Buckling Load in X direction

$n_x$or n = Buckling Coefficient in X direction

ω = Fundamental Natural Frequency of Vibration

f or f = Coefficient of Free Vibration

f_p= Coefficient of Free Vibration in term of P

f_s= Coefficient of Free Vibration in terms of S

D = Flexural Rigidity

$w''^{R} = \frac{\partial^2 w}{\partial R^2}$; $w''^{Q} = \frac{\partial^2 w}{\partial Q^2}$; $w''^{RQ} = \frac{\partial^2 w}{\partial R\, \partial Q}$

$k''^{R} = \frac{\partial^2 k}{\partial R^2}$; $k''^{Q} = \frac{\partial^7 k}{\partial Q^2}$; $k''^{RQ} = \frac{\partial^2 k}{\partial R\, \partial Q}$

mab =Final Support Moment (where a,b = 1,2,3,....)

yr ,yq = Coefficients of Slope in R & Q direction

FEM_E and FEM_I =Fixed End Moment for External and Internal strip of continuous plate.

$SPTM_E$ and $SPTM_I$ = Support Moment for External and Internal strip of continuous plate.

$SPNM_E$ and $SPNM_I$ = Span Moment for External and Internal Strip of continuous plate.

ABSTRACT

This research work is aimed at the development of computer programs using MATLAB based on the new polynomial shape functions, for analysis of single panel and continuous thin isotropic rectangular plates. Twelve single panel plate types of different boundary conditions namely SSSS, CCCC, CSSS, CSCS, CCSS, CCCS, SSFS, SCFS, CSFS, CCFS, SCFC and CCFC for aspect ratio, s= b/a, were analyzed for pure bending, buckling and free vibration using computer programs developed in this work. Expressions were derived and MATLAB codes were applied systematically to develop the programs. Furthermore, both one-way and two-ways continuous plates were analyzed for fixed edge and support moments. These continuous plates were divided into single panels, strip sections were taken and analyzed manually using stiffness method, for support and span moments. After which programs were developed for analyzing the continuous plates. For single panel plates, the values of 'u','α' ,'β', 'β_1' 'δ' and 'δ_1' been coefficients of amplitude, deflection, center moments in x- and y- directions, and shear force in x- and y- direction respectively, were obtained for aspect ratio of 1.0, 1.2,1.4, 1.5,1.6 and 2.0, for each plate condition. For instance, for aspect ratio1.0, the values 'u', 'α', 'β', 'β_1' 'δ' and 'δ_1' for SSSS plate obtain are 0.04236, 0.00414, 0.05163, 0.05163, 0.07491, and 0.37491 respectively. Also values of 'n' and '*f*' been coefficients of critical buckling load and fundamental natural frequency respectively were obtained. The values for 'n' and '*f*' for aspect ratio of 1.0 for SSSS plate, are 39.508 and 19.749 respectively. To validate the values, these coefficients were compared with existing literatures, and the percentage differences were insignificant, hence considered adequate. For continuous plate, the results from both the manual and computer programs developed were compared, and were found to have percentage differences of less than 1%, indicating that they are very close to each other. Hence, consider adequate. Therefore, the conclusion that, the developed computer programs are adequate, easy, quicker and accurate way of analyzing Thin Rectangular Single Panel Isotropic Plates and Continuous Plates, and also that, polynomial shape functions are adequate and less cumbersome for analysis of Continuous plates.

Key Words: Ritz equation, Total energy functional, Pure bending, Polynomial series, Shape functions, Critical buckling load, Fundamental frequencies, Single panel plate, Continuous plate, MATLAB and Computer Program.

CHAPTER ONE

INTRODUCTION

1.1 Background of the Study

Plates are straight, plane, two-dimensional structural components, in which one dimension, known as the thickness; ‘h’, is much smaller than the other dimensions (Szilard, 2004). They are bounded geometrically by either straight or curved lines. They are similar to beams, but do not serve as structural components only. They can also form complete structures such as slab-bridge. The two-dimensional structural action of plates, result in lighter structures and offer economic advantage.

Consequently, plates and plate-type structures, have gained special importance and notably increased in applications in recent years (Szilard, 2004). A large number of structural components in engineering structures, can be classified as plates. Some practical examples of plates in civil engineering structures, are bridge deck and slab bridges, floor and foundation slabs, thin retaining walls, etc. Plates are also indispensible in aerospace and ship building industries. They also form machine parts of some mechanical devices.

Plates can be classified as thin or thick plate, based on their thickness, 'h', and different principles are used in analyzing them. They may also be isotropic or orthotropic plate; linear-elastic plate, based on stress-strain relationship defined by Hooke's law, or non-linear elastic; rectangular or circular or triangular or trapezoidal plate; and can be single panel or continuous over supports.

In classical or equilibrium approach, plate problems have been formulated and analyzed by many researchers or scholars such as Navier, Levy and Timoshenko as cited in Ibearugbulem et al. (2012), using trigonometric formulated shape functions. Others went ahead to use energy and numerical methods, due to the difficulties in using classical method to obtain solutions to plates. None of these methods and scholars used Taylor -Maclaurin's series, to represent the deformed shape function in formulation of plate problems, and in the determination of the parameters, such as forces, moment, displacement etc (Ibearugbulem et al., 2012).

With the advent of high-capacity computers, which have opened another window for easy analysis and modeling of plate problems, various computer languages, have been developed, such as MATLAB, FORTRANS, MINITAB, and Object-Oriented Programming Languages such as Java, C, C++, Visual Basic Lanaguages. The applications of these languages in the development of computer programs, have made it imperative to ease the difficulties of designers and researchers. One of such computer software is WinPlate Primer Software, which was developed by Szilard based on finite element method that employs trigonometric shape functions (Szilard, 2004).

The works of Navier and Levy on pure bending of thin rectangular plate with all the four edges simply supported(i.e SSSS plate), which both employed direct integration of the differential equation based on assumed trigonometric shape function, had a percentage difference of 2.4% on the average (Ibearugbulem, 2013). Some others such as Hartman (1991), Ugural (1999), and Ventsel and Krauthammer (2001), went ahead to use approximate methods (i.e energy and numerical approaches) to obtain the solution for the same SSSS plate. However, all the results they obtained were different, even though the differences were marginal for maximum deflection.

Due to the difficulties in using both exact and approximate methods and the differences between the results obtained from both methods, Ibearugbulem et al. (2013) used Taylor-Maclaurin's series to represent the deformed shape function of SSSS plate, and hence obtained results that were comparable to the existing results, even though, it still has minimal difference in maximum deflection. Ibearugbulem (2011), applied this same series and obtain the critical buckling load for the same SSSS plate. Njoku et al. (2013), used this Taylor-Maclaurin's series formulated-shape function in Galerkin's equations, to analyze the free vibration of thin rectangular isotropic CCCC plate, and theresults they obtained were close to those obtained by Ventsel and Krauthammer (2001). It is of no doubt, that the use of Taylor-Maclaurin’s formulated-shape functions, has alleviated greatly the difficulties earlier encountered using both exact and approximate methods.

But, there is dearth of literature in the area of plate analysis using computer programmes based on polynomial formulated-shape function and not to talk of using computer programmes written in Matlab programming language for analyzing plate problems in pure bending, buckling, and free vibration of plate.

Furthermore, there is no evidence at all that Taylor-Maclaurin's series formulated shape functions, has been applied in the analysis of continuous plates spanning in one way and continuous plates spanning in two ways. In addition, there is no evidence of the existence of any computer software program using Matlab for such analysis.

Hence, this research work, encompasses analysis and development of computer programs written in Matlab programming language for analyzing thin isotropic linear-elastic single panel and continuous rectangular plates using polynomial series formulated -shape functions in Ritz energy equation. It involves the analysis and programming of twelve different single panel plate cases based on their edge conditions so as to determine the deflection, moments, and shear forces. Also, the programs analysis continuous plate spanning in one way and spanning in two ways to obtain fixed edge moment. Stiffness method of analysis, was dopted in the analysis of the continuous plate for the support and span moments.

1.2 Statement of the Problem

Analysis of single panel and continuous rectangular plates using both classical and approximate methods, is too rigorous and time consuming, which results in most times, to computational errors. Hence, there is the need for a simplified approach to rectangular plate analysis by developing interactive computer programs to help analysts do their work with less time and effort. This work, is concerned with the development of computer programs written in Matlab language for pure bending, stability and free vibrational analysis of single panel rectangular isotropic plates and continuous rectangular plates.

In addition, Szilard (2004), noted that classical analysis of continuous plate - with the exception of the simplest cases- is quite cumbersome. Also, Timoshenko and Woinowsky-Krieger(1959) observed that the application of the rigorous classical methods to the design of continuous plates or floor slabs, often leads to cumbersome calculations and that the accuracy of results obtained, is illusory on account of many more or less indeterminable factors affecting the magnitude of the moments of the plate. It is seen clearly from the above statements by eminent scholars in this field that, continuous plate analysis based on manual computation is highly tedious and limited in application to real life problems. Hence, the need to develop computer programs to enhance the analysis of continuous plates is imperative. Therefore, this work is timely and very vital to designers' of plate structures.

1.3 Objectives of the Study

The main objective of this research work, is to develop computer programs for analysis of single panel and continuous rectangular plates. And the specific objectives are as follows:

(i) To develop computer programs for the analysis of single panel rectangular plates in pure bending. In all, twelve(12) different plate cases with various boundary conditions, (namely, SSSS, CCCC, CSSS, CSCS, CCSS, CCCS,SSFS, SCFS, CSFS, CCFS, SCFC and CCFC plates) were treated;

(ii) To develop computer programs for the buckling/stability analysis of the same twelve single panel rectangular plate cases considered above;

(iii) To develop computer programs for the free vibration analysis of the same twelve single panel rectangular plate cases.

(iv) To analyze and develop computer programs for the analysis of continuous plate spanning in one way;

(v) To analyze and develop computer programs for the analysis of continuous plate spanning in two ways.

(vi) To compare the results of the computer programs with other results.

1.4 Justification of the Study

Solving the partial governing differential plate equations manually based on trigonometric assumed shape functions, has not been easy for researchers and practitioners. It is tedious, laborious and time consuming. For this reason, many people, have not benefited from the numerous advantages of plate structures. But, the computer programs developed in this work, will simplify the analysis of thin isotropic rectangular plates and give quick analysis of plates. This will therefore, save time and effort spent in obtaining useful results, unlike the traditional tedious approaches earlier mentioned.

Furthermore, the use of plates in aerospace and ship building, is indispensible because of their light weight and their ability to be folded easily to various shapes. The programs develop in this work would open a new window for more programs to be developed (based on polynomial formulated shape functions) to simplify more complex plate analysis in this areas.

1.5 Scope of the Study

In this work, the analysis and computer modeling of pure bending of thin, isotropic, linear elastic single panel rectangular plate case with twelve different boundary/ edge conditions (namely, SSSS, CCCC, CSSS, CSCS, CCSS, CCCS,SSFS, SCFS, CSFS, CCFS, SCFC and CCFC) were treated. In addition, the buckling/stability analyses of thin isotropic linear elastic single panel rectangular plate cases with twelve different boundary/ edge conditions were carried out.

Also, covered in this project, are free vibration analyses of thin isotropic linear elastic single panel rectangular plate cases with same twelve different boundary/ edge conditions as above.

Finally, the analysis and computer modeling of continuous plates spanning in one way and continuous plates spanning in two ways, were treated.

CHAPTER TWO

LITERATURE REVIEW

2.1 Pure Bending Analysis

Pure bending analysis of rectangular plate, has been carried out by various researchers in the past, among them are Timoshenko and Woinowsky-Krieger as cited by Ibearugbulem et al. (2012), who gave a great insight into plate analysis. They laid the solid foundation of the traditional classical approach, which uses trigonometric shape functions in plate analysis. They analyzed rectangular plates with various boundary conditions under various load applications and obtained results which serve as references in theory of plate today.

Pure bending of plate involves a combination of bending in two perpendicular directions. There are various methods or approaches used for the analysis of rectangular plate in bending; examples are classical or equilibrium method, energy method and numerical method. Szilard (2004) reported that classical solutions of the partial differential equation of plates, are far more difficult. He further stated that, one can transform this partial differential equation into ordinary differential equation by separating the variables.

Ventsel and Krauthammer (2001) analyzed a rectangular plate using classical approach by assuming trigonometric shape functions. They began the application of the development of plates bending theory with the thin rectangular plates. They reiterated that, thin rectangualar plates represent an excellent model for the development of plates and serve as a check of various methods for solving the governing differential equation. In their work, they considered some mathematically exact solutions in the form of double and single trigonometric series applied to rectangualar plates with various types of supports and transverse loads, plates on elastic foundation and continuous plates. Also, according to Szilard (2004), a mathematically exact stress analysis of a thin plate- subjected to loads acting normal to its surface requires solution of the differential equations of three dimensional elasticity. Such method however, mostly lead to insurmountable mathematical difficulties.

Considering the unique position classical plate theory occupies on the subject of plates. The formulation is in term of transverse deflection w(x, y), for which the governing differential equation is the fourth order, which need only two boundary conditions to satisfy at each edge. In analyzing plate bending problems, Szilard (2004) added that, there are four types of mathematically exact solutions available for plate problems, namely, closed –form solution, solution of the homogenous biharmonic equation upon which a particular solution of the governing differential equation of the plate is superimposed, double trigonometric series solution, and single series solution.

From the works of Navier and Levy on pure bending analysis of thin rectangular plate with all the four edges simply supported (i.e SSSS plate), using direct integration of the differential equation based on assumed trigonometric shape function, Ibearugbulem et al. (2013) found out that, the difference between their results was 2.4% average. Ventsel and Krauthemmer (2001), expantiated on Navier's solution of the governing differential equation, which was developed based on double trigonometric series for SSSS plate. According to them, the solution of the governing differential equation, that is, the expressions of the deflected surface, w(x, y), and the distributed surface load, P(x, y), have to be sought in the form of an infinite Fourier series. They further submitted that, the infinite series solution for the deflection generally, converges quickly; thus satisfactory accuracy can be obtained by considering only a few terms. They added that, since the stress resultants and couples are obtained from the second and third directives of the deflection w(x, y), the convergence of the infinite series expressions of the internal forces and moments, is slow, especially, in neigbhourhood of the plate edges. This slow convergence, they said is also accompanied by some loss of accuracy in the process of calculation. They therefore, suggested that, one can improve the accuracy of solutions and convergence of series expressions of the stress resultants and couples by considering more terms in the expressions and by using a special technique for an improvement of the convergence of Fourier's series. Due to the unsatisfactory bending moments and shear forces calculations from Navier's approach, Ventsel and Krauthemmer (2001), commented that, Levy's single Fourier series approach, is more practical, because it is easier to perform numerical calculations for single series than double series, and its applicability to plates covers various boundary conditions.

Apart from the rigorous classical approach, which is almost extensively Newton's law of equilibrium of forces in the development of differential equations for plates, the approach based on Bernoulli's principle of virtual work which replaces the force vectors by work and potential energy can be used. Therefore, some researcher such as Hartman (1991), Ventsel and Krauthammer (2001), Ugural (2003), and Szilard (2004), used approximate methods (i.e energy and numerical approaches), based on trigonometric assumed shape functions, to obtained solution for the same plate with all the four edges simply supported (i.e SSSS) plate. But, all their results varied even though the differences were marginal, at the maximum deflection.

According to Szilard (2004), energy methods may be preferable to the more rigorours classical solutions, for the fact that they are easier, both conceptually and mathematically; they are extremely powerful to obtain reusable analytical solutions, even for plates of arbitrary shape and boundary conditions, and finally, they provide a valuable preparation for understanding the principle of finite element methods, which have rapidly become the most dominant numerical method in structural analysis.

But, Ibearugbulem et al.(2013) analyzed a transversely loaded thin rectangular SSSS (i.e plate with all the four sides simply supported) plate by theoretically formulating the shape's function based on Taylor-Maclaurin's series, which they substituted in Ritz energy equation to obtained deflection. The values of deflection coefficient obtained from the work when compared with other research works, gave an average percentage difference of about 3.63; and this indicates that Taylor-Maclaurin's shape function is very close to the exact displacement shape function of SSSS plate. They concluded that the Taylor-Macaurin's shape function, is a close approximation of the exact displacement function of the deflected simply supported rectangular thin plate.

Soon after, Ibearugbulem et al.(2014) carried out pure bending analysis of two rectangular thin plates, both having their three edges simply supported and the other one edge clamped or fixed (i.e SCSS and CSSS), but with the two plates having different orientation or platform, using a new approach. They carried out this by work principle, which requires substituting the formulated polynomial shape functions into the work error equation to obtain the results of deflection and moments of the plates. Wu et al. (2014), studied pure bending using finite plate analysis based on a nodal integration approach (i.e exact nodal-averaged shear strain method). They carried out this by finite element analysis of Reissner-Mindlin plates, in which a combination of the shear interpolation method for the plate element with an area-weighted averaging technique for the nodal integration of shear energy to relieve shear locking in the thin plate analysis as well as to pass the pure bending patch test.

Ibearugbulem et al. (2013), recommended the use of direct integration and work principle as a new approach in pure bending analysis of isotropic rectangular plates.They analyzed SSSS rectangular plate by direct integration of the governing differential equation of isotropic rectangular plate. The shape function obtained was expressed in the form of Taylor series.

Furthermore, Ibearugbulem et al. (2014), applied Ritz, Galerkin and work error methods in the derivation of pure bending equations. From the three methods used, the amplitude of deflection equation, which is a constant of the equation, for each method was obtained, and when they substituted the values of the shape function into each expression for the amplitude of the three methods, the results obtained, were the same. Also, the results obtained for deflection, edge and central moments, and edge shear force, were the same for the various aspect ratios used. They observed that, the range of aspect ratios in which the assumption of plate behavior based on energy approach, becomes valid is $1 \leq s \leq 2.559461$, where 's' is the aspect ratio, is represented as b/a. Hence, for ultimate limit state design, they proposed the safe range of aspect ratios to be $1 \leq s \leq 2.5$.

In addition, Ibearugbulem (2014), used the product of two mutually perpendicular truncated polynomial series as shape function for rectangular plate analysis. He carried out this by truncating the polynomial series at the fifth term in order to satisfy both the kinematic and kinetic boundary conditions. He opined that, the aim of this work, was to adopt this function as a very good approximation of the deflection function for first mode analysis of plate continuum. The results obtained from his work, was compared with the work of Timoshenko and Woinowsky-Krieger (1959) for aspect ratios 1 to 2, and the maximum percentage difference was 4.28 at aspect ratio of 2. He concluded that, the obtained data from using this truncated series function in energy methods, are very close to the data obtained from numerical and other methods.

Polynomial shape functions have some how, ease the rigorous computation of plate bending analysis to some extent. However, there is still much time spent in manual computation. Therefore, there is a need to develop computer programs based on polynomial shape functions to aid in pure bending analysis of rectangular plates

2.2 Buckling or Stability Analysis

Nearly all structural materials have the tendency to buckle under loads, depending on the direction and magnitude of such applied loads. Various shapes of thin plates used in naval and aerospace structures, are subjected to normal compressive and shearing loads acting in middle plane of the plate considered. Plate buckling is of great practical impotance. The effect of variable in-plane forces on the buckling of rectangular plates, has been recently studied by several authors, such as, Szilard (2004), Iyenger (1988), etc. All these studies were based on the classical thin plate theory.

According to Ventsel and Krauthammer (2001), the buckling load depends on the plate thickness, and in many cases, a failure of the plate elements, may be attributed to an elastic instability and not to the lack of their strength. They further stated that, the stability analysis of plate, is qualitatively similarly to the Euler stability analysis of column. In addition, they defined plate buckling as, the transition of plate from the stable state of equilibrium to the unstable one, and, the smallest value of the load producing buckling as the critical or buckling load.

Iyenger (1988), opined that the basic difference between a column and a plate, lies in the buckling characteristic. A column, once it buckles, cannot resist any additional axial load. Thus, the critical load of the column is also its failure load, while a plate, based on edge supports, continuous to resist the additional axial load even after 10-15 times the buckling load. This to him, means the postbuckling load (elastic) for plate is higher, and should be utilized when designing structural members to minimize the weight of the structure. He investigated thin rectangular plates using both equilibrium and energy approaches.

Kang and Leissa (2001) presented the results for buckling factors of SS-F-SS-F plate, loaded by unidirectional in-plane moments. These results were very close to those obtained in existing literatures. Later on, Leissa and Kang (2002) extended the study to SS-C-SS-C plate under the same loading type. The stability equation they obtained for the thin plate, can be separated in the two directions, as a product of two one-variable functions, and the solution obtained was exact. But, Bert and Devarakonda (2003) determined the buckling factors of rectangular plates with nonlinear in-plane stress distribution. They presented solution for the in-plane pre-buckling stress distributions in series form, and solved thin plate buckling equation only for simply supported plate.

Shutrin and Eisenberger (2005), investigated the buckling of plate with variable in-plane forces, and found out that shear deformations, have significant effect on the stability of plates, and so must be included in the derivation of the buckling equations. Azhari *et al.* (2000) used the spline finite strip method, and they discovered that by adding bubble functions, they were able to improve on the results of buckling analysis of plates. They presented approximate solutions for two combinations of boundary conditions only.

Also, Kang and Shim (2004) extended the solution for plates with two opposite edges simply supported and any other boundary conditions on the other two edges. Romeo and Ferrero (2001) reported results of the analysis of anisotropic rectangular plates with bi-directional in-plane moment loading. They solved the buckling equation using the Rayleigh–Ritz method, by assuming beam vibration modes in the two directions and minimizing the total energy of the plate. Their results are approximate.

Others who work on plate buckling using the trigonometric shape functions, are Szilard (2004) and Ventsel and Krauthammer (2001). They stated that, in mathematical formulation of elastic stability problems, neutral equilibrium is associated with the existence of bifurcation of the deformations. According to this formulation they said, the critical load can be identified with the load corresponding to the bifurcation of the equilibrium states. In other words, the critical load, is the smallest load at which both the flat equilibrium configuration of the plate and slightly deflected configuration, are possible.

Some researchers recently took a different approach from the previous by using Taylor-Maclaurin’s series to ease the stress of the former. For example, Ibearugbulem et al. (2011), carried out buckling analysis of axially compressed thin rectangular SSSS plate using Taylor-Maclaurin's shape function in Ritz method. The results they obtained were very close to those obtained from the use of trigonometric shape function in of classical methods. He discovered that the buckling load decreased with increase in aspect ratio. Also, Ibearugbulem et al. (2012), carried out instability analysis of axially compressed CCCC thin rectangular plate using Taylor -Maclaurin's series shape function in Ritz method. The results obtained were satisfactory when compared with those available in existing literatures. Furthermore, Ibearugbulem et al. (2012), applied direct variational principle in elastic stability of rectangular flat thin plate.

In addition, Onwuka et al. (2013), carried out plastic buckling of thin rectangular SSSS plates subjected to uniaxial compression using Taylor-Maclaurin's shape function. From their result, they concluded that the use of Taylor Maclaurin's shape functions is adequate in plastic analysis of SSSS plates. Ezeh et al (2013), investigated the use of polynomial shape functions in the buckling analysis of CCFC rectangualar plate. They approach this by obtaining a peculiar shape function, which they substituted in Ritz energy equation to obtain the critical buckling load. They plotted the graph of critical buckling load against aspect ratio. It was discovered that for aspect ratios of 0.4, 0.5 and 1.0, the critical buckling loads coefficients were 26.94, 17.39 and 4.83. It was also observed from the behavior of the graph that as aspect ratio increased from 0.1 to 2.0, the critical buckling load decrease.

Also, Ezeh et al. (2014a), worked on elastic stability analysis of a clamped thin rectangular flat plate using Galerkin's indirect variational method. Their results were compared adequately with established results. In addition, Ezeh et al (2014), investigated the behavior of buckled CSFS isotropic rectangular plate using polynomial series shape function in Ritz method. From the graph of crtical buckling load against aspect ratio, it was observed that as the aspect ratio increases from 0.1 to 2.0, the critical buckling load decreases. For aspect ratio of 0.2, 0.4, 0.8, 1.0, the values of non dimensional parameters of buckling load were 25.35, 6.61, 2.08, and 1.63 respectively. It was discovered that for aspect ratios of 0.2, 0.4, 0.8, the percentage differences between the critical buckling load obtained from their study and those of Iyenger and Ibearugbulem are -0.00394%, 4.3217%, -3.9352% and 37.2442%, 8.6043%, -14.6256% respectively.

Moreso, Ibearugbulem et al (2014), analyzed isotropic SSFS rectangular plate using polynomial shape function for buckling. Their result show that the percentage difference beween the critical buckling load in their work for aspect ratios of 0.5 and 1.0 and those of Timoshenko and Ibearugbulem, were -2.9576%, -5.4079% and -8.7594%, -13.672% respectively. Eziefula et al. (2014), performed analysis on plastic buckling of an isotropic C-SS-SS-SS plate under in-plane loading using Taylor's series. Ibearugbulem et al (2015), carried out inelastic stability analysis of uniaxially compressed flat rectangular isotropic CCSS plate. The results obtained compared favourably with the elastic stability values and percentage differences ranged from -0.353% to 7.427%. Therefore, they concluded that, the theoretical approach proposed in the study, is recommended for the inelastic stability analysis of thin flat rectangular isotropic plates under uniform in-plane compression.

From all the results obtained, based on the use of Taylor Maclaurin's formulated shape functions in energy equations, it is observed that the percentage differences between these results and those obtained by classical methods using trigonometric shape functions, are all within the acceptable limit in statistics, and that these results, are both lower and upper bound solutions in some case to those obtained by classical approaches.

To further investigate the suitability of Taylor-Maclaurin's formulated shape functions in plate analysis, Ezeh et al. (2014), performed stability analysis of orthotropic reinforce concrete shear wall using the new shape function. They results obtained, were very close to those obtained by the use of trigonometric shape functions. This further confirms the suitability of Taylor-Maclaurin's or polynomial formulated shape functions for plate analysis.

This new approach, no doubt has reduced the stress of computation, but the approach is still time consuming to an extent. Hence, this work is concerned with the development of computer programs based on polynomial shape functions for easy and quick bending, vibrational and buckling analysis of thin rectangular plates.

2.3 Free Vibration Analysis

Free vibration of rectangular plate can be studied by specifying the boundary conditions of the plate. Szilard (2004) used single degree of freedom (DOF) system to analyze for the deflection of SSSS plate loaded uniformly and the results, obtained were comparable to Navier's solution. He stated that, the undraped free flexural vibration of rectangular plates, are basically boundary value problems of mathematical physics. He also, used energy and numerical methods to obtained natural frequencies of thin rectangular plates. In addition, he stated that, damping effects are caused either by internal friction or the surrounding media. He proceeded by saying that, although structural damping, is theoretically present in all plate vibrations, it has usually little or no effect on the natural frequencies and steady –state amplitudes.

Also, Ventsel and Krauthammer, (2001) and Chakreverty (2009), who took classical and energy approaches, used trigonometric shape function to obtained natural frequency of SSSS and CCCC plates. According to Ventsel and Krauthammer, (2001), free vibration occurs in the absence of applied loads, but may be initiated by applying initial conditions to the plates. The free vibration deals with natural characteristics of the plates, and these natural vibrations occur at discrete frequencies, depending only on the geometry and material of the plates. They added that the most important part of the free flexural vibrations of plates, is to determine the natural frequencies and the mode shapes of the vibration associated with each natural frequency. Kanak and Abir (2013), worked on free vibration of isotropic and composite rectangular plates, in which finite element formulation in ANSYS computer package, was used to analyze square plates of various boundary conditions (namely, SSSS, SSSC, SCSC, SCCC and CCCC) and different thickness ratios.

These authors carried out their work based on the use of trigonometric shape functions, which are mostly assumed, and make the process of computation some how iterative in most cases; thereby making the computation more difficult for analysts involved in plate analysis.

In view of these difficulties, and the quest to simplify free vibration of plate analysis, the use of polynomial formulated shape functions, have been investigated by some recent researchers. Among them are, Njoku et al. (2013), who considered free vibration analysis of an all-round fixed (i.e CCCC) thin rectangular isotropic plate using Taylor series based shape function in Galerkin's method. In doing so, he derived a peculiar shape function by applying the boundary conditions of the plate in Taylor series form of the equation, and substituted the initial result in Galerkin's functional to obtain the equation for the fundamental natural frequency of the free vibrating plate.

The results obtained were compared with those of Chakreverty (2009), and they were very close. He also, considered the following plates: plate simply supported on all four edges (SSSS), plates with two adjacent clamped and the other two adjacent, simply supported (CCSS), plates with one edge clamped and other edges simply supported (CSSS), and plates with two opposite edges clamped and the other opposite edges simply supported (CSCS). From the results obtained, a maximum percentage difference of 0.0778%, was observed for SSSS plate for aspect ratio of 0.10, while that for CCCS plate was 0.409%.

Also, Ibearugbulem et al. (2012), performed vibration analysis of thin rectangular SSSS plate using Taylor-Maclaurin's shape function. Soon after that, Ibearugbulem et al. (2013) carried out vibration analysis of CSSF and SSFC panel using same energy method. The percentage differences in the results obtained were minimal and within acceptable statistical limit. They further carried out vibration analysis using finite difference methods for SSSS, CCCC, and CSCS plates. From their results obtained, it was obvious that they were better with increased number of grid points. They concluded that, finite difference method (FDM) is a good numerical method for simplying thin plate analysis.

Ezeh et al. (2014), carried out vibration analysis of a plate with one free edge using energy method. The Taylor–Maclaurin's shape function derived was substituted in potential energy functional to obtain the fundamental natural frequency. Their study brought up a new relationship between fundamental natural frequency and aspect ratios. Also, they developed graphical models which can be used in place of the primary equations. Ebirim et al. (2014), further analyzed free vibration of isotropic rectangular plates (with CSCF and SCFC boundary conditions) using Taylor- Maclaurin's series. It is worth mentioning that, they results obtained based on these series, are satisfactory when compared with those of available literatures, the computational approach, is quite easier and simplified when compared with those based on trigonometric shape functions using classical methods.

From the available literatures, none of these researchers who employed Taylor - Maclaurin's series in the analysis of plates undergoing free vibration, develop or use computer program based on them, to analyze free vibration of thin rectangular plates.

2.4 Continuous Plates

Floor slabs used in buildings and bridges, besides being supported by exterior walls, often have intermediate supports in the form of beams and partitions or columns. Floor slabs are usually sub-divided by their supports into several panels, either of equal or unequal panels. Continuous plates, are externally statically indeterminate. The classical methods used in the analysis of continuous plate fall into two distinct categories, namely force and deformation methods.

Szilard (2004), noted that classical analysis of continuous plate, with the exception of the simplest cases, is quite cumbersome. He stated further that, numerical or engineering approaches, are used almost exclusively in the praxis to obtain usable approximate solutions of these important plate problems in an economical way. Timoshenko and Woinowsky-Krieger (1959), observed that the application of the rigorous classical methods to the design of continuous plates or floor slabs, often, leads to cumbersome calculations and that the accuracy of results obtained, is illusory on account of many more or less indeterminable factors affecting the magnitude of the moments of the plate. These includes flexibility and the torsional rigidity of the supporting beams, the restraining effect of the surrounding walls, the anisotropy of the plate itself, and the accuracy in estimating the value of such constants as the Poisson ratio, v.

Assuming the intermediate support as simply supported, Szilard (2004) used force method to analyze simply supported continuous plate in one direction (x - direction). He performed this analysis based on trigonometric shape functions substituted in moment equations, and obtained the central moments at each panel.

Timoshenko and Woinowsky-Krieger (1959), analyzed simply supported continuous plates with intermediate beams and continuous plates without intermediate beams (that is flat slabs). They further considered approximate design of continuous plate with equal spans in two-directions. On his own, Szilard (2004) analyzed only continuous plate with rigid intermediate supports (that is, plate with zero lateral deflections at the support) by assuming that the supports do not prevent rotations of the plate. Ventsel and Krauthammer (2001), studied a 3-span continuous plate simply supported using force method.

In analysis of continuous plates, most efforts, have been on classical and approximate methods. Timoshenko and Woinowsky-Krieger (1959), stated that for classical method, the procedure of calculation, can be simplify by restricting the Fourier series representing bending moment in the plate to its initial term or by replacing the actual values of moments or slopes along some supports of the plate, with their average values or, finally, by use of a moment distribution procedure.

Again, Szilard (2004) mentioned that, in continuous plate' analysis spanning in two directions, application of the so-called 'seven - moment equation' derived by Galerkin, can be used, which he said, is valid for each intermediate edge. He further stated that, although it is theoretically possible to obtained close-form solutions for continuous plates in two-directions, the mathematical manipulations required, will become extensively prohibitive. He, therefore, recommended the application of numerical approximations, such as finite element method, finite difference method etc, in continuous plate analysis, which he said offered a little simpler approach and valid practical results, and so it can be applied with computer easily to reduce stress of computation.

All these researchers used only trigonometric functions in their work; neither of them used polynomial shape functions nor wrote programs based on the polynomial formulated shape functions in the analysis of continuous plate.

2.5 Computer Application in Plate Analysis

In fast advancing technology of twenty first century, where time has become a resource and with the increasing complexity of scientific and engineering problems, the use of computer has become indispensible in getting certain things done, and done timely. Several computer languages such as MATLAB, FORTRAN, C++, MINITAB etc have been developed to aid programming of problems.

In plate analysis, some of these languages, have been used to developed programs for analysis of plate problems. For example, Szilard (2004) develops WINplate program software based on finite element method. SAP 2000 is also available for plate analysis based on trigonometric shape function. Also, MSC/NASTRAN which has been developed is based on trigonometric approach. The Dlubal Program Plate-Buckling, analyzed plate with or without stiffeners by successive eigenvalue calculation of the ideal buckling values for the individual plate stress condition as well as of buckling value for the simultaneous effectives of all stress components.

Throughout this review, it is very obvious that there is limited information on programs written based on polynomial shape functions using MATLAB for analyzing isotropic continuous rectangular plates, so as to ease the stressful and time consuming effort of plate analysts. Hence, this work is timely and will be a leverage to all involved in plate analysis.

Matlab is an interactive working environment, in which the user can carry out numerous complex computational tasks with few commands. It is an acronym for Math's Laboratories. It offers a nice combination of handy programming features with powerful built-in numerical capabilities, and its M-files programming environment, allows for implementation of moderately complicated algorithms in a structured and coherent fashion, and can solve more difficult problems without trying to reinvent the wheels (Chapra, 2012). Matlab is a high-level software package with many built-in functions that make the learning of numerical methods, much easier and more interesting (Yang et al., 2005).

Also, according to Wilson et al. (2003), Matlab embodies an interactive environment with a high-level programming language, supporting both numerical and graphical commands for two- and three - dimensional data analysis and presentation. He further said that, its (Matlab) wealth of intrinsic mathematical commands to handle matrix algebra, Fourier series, differential equations and complex-valued functions, and make simple calculator operations of many tasks previously requiring subroutine libraries with cumbersome argument lists, makes it an excellent alternative to languages such as FORTRANS or C++.

CHAPTER THREE

METHODOLOGY

3.1 Materials used

The following materials were used to carry out this project work namely:

- Deflected Shape functions
- Total Potential Engergy Functionals
- MATLAB Software

3.2 Development of Computer Program

The programs were developed with Matlab software based on some derived expressions using the deflected shape functions in Ritz energy equation. Hence, before proceeding into the development of the programs, it is important to first establish the theory and basic principles on which these programs will be based for clarity purpose.

3.3 Single Panel Rectangular Isotropic Plate

The shape function of each of the twelve (12) plate cases under consideration, were derived by assuming a deflected shape function (w) in form of Taylor-Maclaurin's series for a plate subject to a uniformly distributed load, q(kN/m). The series was truncated at the fifth term. This deflection was consider in x- and y- directions using non-dimensional parameters R and Q (where R = x/a and Q = y/b).

The shape function in terms of the non-dimensional parameter, R in x-direction is given by:

$$w^R = a_0+a_1R+a_2R^2+a_3R^3+a_4R^4 \quad (3.1)$$

where a_o, a_1, a_2, $\ldots a_n$ are coefficients of the polynomial.

And for non-dimensional parameter, Q in y-direction, the deflected shape function, w^Q, is given as follows:

$$w^Q = b_0+b_1Q+b_2Q^2+b_3Q^3+b_4Q^4 \quad (3.2)$$

Where b_o, b_1, b_2, …b_n are coefficients of the polynomial.

Then, the different boundary conditions for each of the twelve plate cases were applied to Eqns (3.1) and (3.2) to obtain the values of the coefficients. Thereafter, the deflected shape function (w) for each plate was then obtained by multiplying Eqns (3.1) and (3.2). The product is in the form of Eqn (3.3).

$$w = w^R*w^Q = A(c_1R^{f1}-d_1R^{m1}+e_1R^{n1})\ (c_2Q^{f2}-d_2Q^{m2}+e_2Q^{n2}) \quad (3.3)$$

where c_1,c_2,f^1,f^2,d_1,d_2,m^1,m^2,e_1,e_2 and n^1,n^2 are coefficient of the deflected shape function of each plate, whose values are 0.5, 1, 2, … as the case may be.

And A, is the amplitude of deflection of the equation.

Let $k_i = (c_1R^{f1}-d_1R^{m1}+e_1R^{n1})(c_2Q^{f2}-d_2Q^{m2}+e_2Q^{n2})$ (3.4)

Which is the shape function of each plate.

If $U = (c_1R^{f1}-d_1R^{m1}+e_1R^{n1})$ (3.4a)

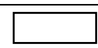
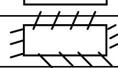
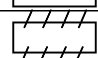
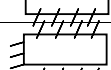
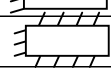
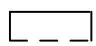
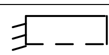
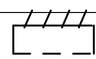
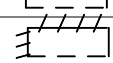
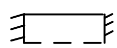
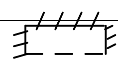
And $V = (c_2Q^{f2}-d_2Q^{m2}+e_2Q^{n2})$ (3.4b)

Then, $k_i = U*V$ (3.4c)

Therefore, $w = Ak_i$ (3.5)

Table 3.1 shows the deflected shape functions for the twelve (12) plates.

Table 3.1 Formulated Polynomial Shape Functions

S/N	Types of Plates	Plate Sketch	Shape Function $W_i = Ak_i$ (where $k_i = k$)	Shape Parameter $k_i = U*V$ (where i = 1,2,3, ..., 12)
1	SSSS		$W = A\,k_1$	$k_1 = (R-2R^3+R^4)(Q-2Q^3+Q^4)$
2	CCCC		$W = A\,k_2$	$K_2 = (R^2-2R^3+R^4)(Q^2-2Q^3+Q^4)$
3	CSSS		$W = A\,k_3$	$K_3 = (R-2R^3+R^4)(1.5Q^2-2.5Q^3+Q^4)$
4	CSCS		$W = A\,k_4$	$K_4 = (R-2R^3+R^4)(Q^2-2Q^3+Q^4)$
5	CCSS		$W = A\,k_5$	$K_5 = (1.5R^2-2.5R^3+R^4)(1.5Q^2-2.5Q^3+Q^4)$
6	CCCS		$W = A\,k_6$	$K_6 = (1.5R^2-2.5R^3+R^4)(Q^2-2Q^3+Q^4)$
7	SSFS		$W = A\,k_7$	$K_7 = (R-2R^3+R^4)(\frac{7}{3}Q-\frac{10}{3}Q^3+\frac{10}{3}Q^4-Q^5)$
8	SCFS		$W = A\,k_8$	$K_8 = (1.5R^2-2.5R^3+R^4)(\frac{7}{3}Q-\frac{10}{3}Q^3+\frac{10}{3}Q^4-Q^5)$
9	CSFS		$W = A\,k_9$	$K_9 = (R-2R^3+R^4)(2.8Q^2-5.2Q^3+3.8Q^4-Q^5)$
10	CCFS		$W = A\,k_{10}$	$K_{10} = (1.5R^2-2R^3+R^4)(2.8Q^2-5.2Q^3+3.8Q^4-Q^5)$
11	SCFC		$W = A\,k_{11}$	$k_{11} = (R^2-2R^3+R^4)(\frac{7}{3}Q-\frac{10}{3}Q^3+\frac{10}{3}Q^4-Q^5)$
12	CCFC		$W = A\,k_{12}$	$k_{12} = (R^2-2R^3+R^4)(2.8Q^2-5.2Q^3+3.8Q^4-Q^5)$

where S-Simply supported edge; C- Clamped edge and F-Free edge

3.3.1 Pure Bending Analysis

The total potential energy functional, Π, of a rectangular plate subject to pure bending as given by Ibearugbulem (2012) and Ventsel et al. (2001), is as follows:

$$\Pi = \frac{Da}{2b^3}\iint \left[\frac{1}{P^4}(w''^{R})^2 + \frac{2}{P^2}(w''^{RQ})^2 + (w''^{Q})^2 \right] \partial R \partial Q - abq\iint w \partial R \partial Q \qquad (3.6)$$

where D = flexural rigidity of the plate,

a and b are plate dimensions in x- and y- directions respective.

P is aspect ratio a/b, and

q is the applied uniformly distributed load.

And $w''^R = \frac{\partial^2 w}{\partial R^2}$; $w''^Q = \frac{\partial^2 w}{\partial Q^2}$; $w''^{RQ} = \frac{\partial^2 w}{\partial R\,\partial Q}$.

In order to make Eqn (3.6) usable for this study, then Eqn (3.5) was substituted into Eqn (3.6) within integral 0 to 1, to obtain Eqn (3.7)

$$\Pi = \frac{Da}{2b^3} A^2 \iint \left[\frac{1}{p^4} (k''^R)^2 + \frac{2}{p^2}(k''^{RQ})^2 + (k''^Q)^2 \right] \partial R \partial Q - abqA \iint k \partial R \partial Q \qquad (3.7)$$

where p = a/b, and $k = k_i$, $k''^R = \frac{\partial^2 k}{\partial R^2}$; $k''^Q = \frac{\partial^2 k}{\partial Q^2}$; $k''^{RQ} = \frac{\partial^2 k}{\partial R\,\partial Q}$

Minimizing Eqn (3.7) and making A the subject of the formula yields

$$A = \frac{\int\int k \partial R \partial Q}{\int\int \left[\frac{1}{p^4} (k''^R)^2 + \frac{2}{p^2}(k''^{RQ})^2 + (k''^Q)^2\right] \partial R \partial Q} * \frac{qb^4}{D} \qquad (3.8)$$

$$\text{let } u_p = \frac{\int\int k \partial R \partial Q}{\int\int \left[\frac{1}{p^4} (k''^R)^2 + \frac{2}{p^2}(k''^{RQ})^2 + (k''^Q)^2\right] \partial R \partial Q} \qquad (3.9)$$

Substituting Eqn (3.9) into Eqn (3.8) gives Eqn (3.10)

$$A = u_p * \frac{qb^4}{D} \qquad (3.10)$$

Hence, the deflection 'w' = Ak = $u_p * \frac{qb^4}{D}$ k (3.11)

The Eqn (3.11) is the general deflection expression for thin isotropic rectangular plate given in terms of aspect ratio, p = a/b.

But if s = b/a = 1/p (3.12)

Then, substituting Eqn (3.12) into Eqn (3.8) , yields Eqn (3.13)

$$A = \frac{\int\int k \partial R \partial Q}{\int\int \left[(k''^R)^2 + \frac{2}{s^2} (k''^{RQ})^2 + \frac{1}{s^4} (k''^Q)^2\right] \partial R \partial Q} * \frac{qa^4}{D} \qquad (3.13)$$

$$\text{Let } u_s = \frac{\int\int k \partial R \partial Q}{\int\int \left[(k''^R)^2 + \frac{2}{s^2} (k''^{RQ})^2 + \frac{1}{s^4} (k''^Q)^2\right] \partial R \partial Q} \qquad (3.14)$$

Substituting Eqn (3.14) into Eqn (3.13), yields Eqn (3.15)

$$A = u_s * \frac{qa^4}{D} \tag{3.15}$$

Hence, the deflection for each plate is given by Eqn (3.16)

$$w = u_s * \frac{qa^4}{D} k \tag{3.16}$$

Then, $w = \alpha \frac{qa^4}{D}$ (3.17)

where $\alpha = u_s * k$, and u_p and u_s are the amplitude coefficients given in terms of aspect ratios, p and s respectively.

The Eqn (3.16) or (3.17) is the general deflection expression for thin isotropic rectangular plate given in terms of aspect ratio s = b/a.

But, the moment, M, at the mid span of each plate, was obtained from the following equations:

$$M_x = -D\left(\frac{\partial^2 w}{\partial x^2} + v \frac{\partial^2 w}{\partial y^2} \right) \tag{3.18}$$

$$M_y = -D\left(v\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} \right) \tag{3.19}$$

Substituting for x and y in terms of non-dimensional parameters, R = y/a and Q = y/b into Eqns (3.18) and (3.19), yields Eqns (3.20) and (3.21) respectively

$$M_x = -D\left(\frac{\partial^2 w}{a^2\, \partial R^2} + v\frac{\partial^2 w}{b^2\, \partial Q^2} \right) \tag{3.20}$$

$$M_y = -D\left(v\frac{\partial^2 w}{a^2\, \partial R^2} + \frac{\partial^2 w}{b^2\, \partial Q^2} \right) \tag{3.21}$$

Substituting Eqns (3.5) and (3.15) into Eqns (3.20) and (3.21), gives Eqns (3.22) and (3.23) respectively.

$$M_x = -u_s qa^2 \left(\frac{\partial^2 k}{\partial R^2} + v \frac{\partial^2 k}{S^2 \partial Q^2} \right) \tag{3.22}$$

$$M_y = -u_s qa^2 \left(v \frac{\partial^2 k}{\partial R^2} + \frac{\partial^2 k}{S^2 \partial Q^2} \right) \tag{3.23}$$

$$\text{let } \beta = -u_s \left(\frac{\partial^2 k}{\partial R^2} + v \frac{\partial^2 k}{S^2 \partial Q^2} \right) \tag{3.24}$$

$$\text{And } \beta_1 = -u_s \left(v \frac{\partial^2 k}{\partial R^2} + \frac{\partial^2 k}{S^2 \partial Q^2} \right) \tag{3.25}$$

Substituting Eqn (3.24) into Eqn (3.22), and Eqn (3.25) into Eqn (3.23), yields respectively;

$$M_x = \beta\, qa^2 \tag{3.26}$$

$$M_y = \beta_1\, qa^2 \tag{3.27}$$

The shear force in x- and y- directions, were obtained fromthe following Eqns (3.28) and (3.29)

$$V_x = -D\left[\frac{\partial^3 w}{\partial x^3} + (2-v) \frac{\partial^3 w}{\partial x\, \partial y^2} \right] \tag{3.28}$$

$$V_y = -D\left[\frac{\partial^3 w}{\partial y^3} + (2-v) \frac{\partial^3 w}{\partial x^2\, \partial y} \right] \tag{3.29}$$

Substituting for x and y in terms of non-dimensional parameters, R = x/a and Q = y/b into Eqns (3.28) and (3.29), yields Eqns (3.30) and (3.31) respectively

$$V_x = -D\left[\frac{\partial^3 w}{a^3\, \partial R^3} + (2-v) \frac{\partial^3 w}{ab^2\, \partial R\, \partial Q^2} \right] \tag{3.30}$$

$$V_y = -D\left[\frac{\partial^3 w}{b^3\, \partial Q^3} + (2-v) \frac{\partial^3 w}{a^2 b\, \partial R^2\, \partial Q} \right] \tag{3.31}$$

Substituting Eqns (3.5) and (3.15) into Eqns (3.30) and (3.31), gives Eqns (3.32) and (3.33) respectively

$$V_x = -u_s qa\left[\frac{\partial^3 k}{\partial R^3} + (2-v)\frac{\partial^3 k}{S^2\, \partial R\, \partial Q^2}\right] \quad (3.32)$$

$$V_y = -u_s qa\left[\frac{\partial^3 k}{S^3\, \partial Q^3} + (2-v)\frac{\partial^3 k}{S\, \partial R^2\, \partial Q}\right] \quad (3.33)$$

$$\text{Let } \delta = -u_s\left[\frac{\partial^3 k}{\partial R^3} + (2-v)\frac{\partial^3 k}{S^2\, \partial R\, \partial Q^2}\right] \quad (3.34)$$

$$\text{And } \delta_1 = -u_s\left[\frac{\partial^3 k}{S^3\, \partial Q^3} + (2-v)\frac{\partial^3 k}{S\, \partial R^2\, \partial Q}\right] \quad (3.35)$$

Therefore, by substituting Eqn (3.34) into Eqn (3.32) and Eqn (3.35) into Eqn (3.33), the expressions for shear force in the x- and y- directions gives Eqn (3.36) and Eqn (3,37) repectively:

$$V_x = \delta qa \quad (3.36)$$

$$V_y = \delta_1 qa \quad (3.37)$$

Based on the derived equations, programs were developed for pure bending analysis in section 3.5.

3.3.2 Buckling/Stability Analysis

Ibearugbulem (2011) gave the total potential energy functional, Π_x, for a rectangular thin isotropic plate subject to in-plane load in x-direction as:

$$\Pi_x = \frac{D}{2b^2} \iint \left[\frac{b^3}{a^3}(w''^R)^2 + \frac{2b}{a}(w''^{RQ})^2 + \frac{a}{b}(w''^Q)^2\right] \partial R \partial Q - \frac{bN_x}{2a} \iint (w'^R)^2 \partial R \partial Q \quad (3.38)$$

where N_x is the critical buckling load of the plate, and all other parameters retain the same meaning as in section 3.1.1.

For aspect ratio, s = b/a, Eqn (38) becomes Eqn (3.39)

$$\Pi_x = \frac{D}{2a^2} \iint \left[s(w''^R)^2 + \frac{2}{S}(w''^{RQ})^2 + \frac{1}{S^3}(w''^Q)^2\right] \partial R \partial Q - \frac{N_x}{2} \iint s(w'^R)^2 \partial R \partial Q \quad (3.39)$$

Modifying Eqn (3.39) to make it usable in developing the program, requires substituting Eqn (3.5) into Eqn (3.35), to obtained Eqn (3.40)

$$\Pi_x = \frac{DA^2}{2a^2} \iint [s(k''^R)^2 + \frac{2}{S}(k''^{RQ})^2 + \frac{1}{S^3}(k''^Q)^2]\, \partial R \partial Q - \frac{NxA^2}{2} \iint s(k'^R)^2 \partial R \partial Q \quad (3.40)$$

Minimizing Eqn (3.40) and making, N_x the subject of the formula, gives:

$$N_x = \frac{D \int\int [(k''^R)^2 + \frac{2}{S^2}(k''^{RQ})^2 + \frac{1}{S^4}(k''^Q)^2] \partial R \partial Q}{a^2 \int\int (k'^R)^2 \partial R \partial Q} \quad (3.41)$$

The Eqn (3.41) can be rewritten as Eqn (3.42)

$$N_x = \frac{\int\int [(k''^R)^2 + \frac{2}{S^2}(k''^{RQ})^2 + \frac{1}{S^4}(k''^Q)^2] \partial R \partial Q}{\int\int (k'^R)^2 \partial R \partial Q} * \frac{D}{a^2} \quad (3.42)$$

$$\text{Let } n_x = \frac{\int\int [(k''^R)^2 + \frac{2}{S^2}(k''^{RQ})^2 + \frac{1}{S^4}(k''^Q)^2] \partial R \partial Q}{\int\int (k'^R)^2 \partial R \partial Q} \quad (3.43)$$

$$\text{Therefore, } N_x = n_x \frac{D}{a^2} \quad (3.44)$$

where, n_x, is the critical buckling factor or coefficient in x- direction.

The Eqn (3.42) or Eqn (3.44) is the general critical buckling load expression for thin isotropic rectangular plate with aspect ratio s = b/a.

Programs based on Eqns (3.42) and (3.44) were developed for stability or buckling analysis in section 3.5.

3.3.3 Free Vibration Analysis

The total potential energy functional, Π, for free vibration of rectangular Isotropic plate using Ritz method as given by Ibearugbulem (2013) , is as follows:

$$\Pi = \frac{D}{2b^2} \iint [\frac{1}{P^3}(w''^R)^2 + \frac{2}{P}(w''^{RQ})^2 + p(w''^Q)^2] \partial R \partial Q - p\rho h \omega^2 b^2 \iint w^2 \partial R \partial Q \quad (3.45)$$

where ρ is the specific density of the plate, and ω is the fundamental natural frequency or the resonating frequency of the vibrating plate.

Substituting w = Ak, (i.e Eqn 3.5), into Eqn (3.45), yields:

$$\Pi = \frac{DA^2}{2b^2} \iint \left[\frac{1}{p^3} (k''^R)^2 + \frac{2}{p} (k''^{RQ})^2 + p(k''^Q)^2 \right] \partial R \partial Q - p\rho h \omega^2 b^2 A^2 \iint k^2 \,\partial R \partial Q \quad (3.46)$$

Minimizing Eqn (3.46) and making 'ω^2' the subject of the equation, gives Eqn (3.47)

$$\omega^2 = \frac{\int\int \left[\frac{1}{p^4} (k''^R)^2 + \frac{2}{p^2} (k''^{RQ})^2 + (k''^Q)^2 \right] \partial R \, \partial Q}{\int\int k^2 \partial R \, \partial Q} * \frac{D}{\rho h b^4} \quad (3.47)$$

$$\text{Let } f_p^{\,2} = \frac{\int\int \left[\frac{1}{p^4} (k''^R)^2 + \frac{2}{p^2} (k''^{RQ})^2 + (k''^Q)^2 \right] \partial R \, \partial Q}{\int\int k^2 \partial R \, \partial Q} \quad (3.48)$$

$$\text{Therefore, } \omega^2 = f_p^{\,2} * \frac{D}{\rho h b^4} \quad (3.49)$$

$$\text{Hence, } \omega = \frac{f_p}{b^2} \sqrt{\frac{D}{\rho h}} \quad (3.50)$$

The Eqns (3.47) or (3.50) is the general expression for the fundamental natural frequency of a thin isotropic rectangular plate with aspect ratio, p = a/b.

For aspect ratio s = b/a, the Eqn (3.47), becomes:

$$\omega^2 = \frac{\int\int \left[(k''^R)^2 + \frac{2}{s^2} (k''^{RQ})^2 + \frac{1}{s^4} (k''^R)^2 \right] \partial R \, \partial Q}{\int\int k^2 \partial R \, \partial Q} * \frac{D}{\rho h a^4} \quad (3.51)$$

$$\text{Let } f_s^{\,2} = \frac{\int\int \left[(k''^R)^2 + \frac{2}{s^2} (k''^{RQ})^2 + \frac{1}{s^4} (k''^R)^2 \right] \partial R \, \partial Q}{\int\int k^2 \partial R \, \partial Q} \quad (3.52)$$

$$f_s = \left[\frac{\int\int \left[(k''^R)^2 + \frac{2}{s^2} (k''^{RQ})^2 + \frac{1}{s^4} (k''^R)^2 \right] \partial R \, \partial Q}{\int\int k^2 \partial R \, \partial Q} \right]^{1/2} \quad (3.53)$$

From Eqn (3.51), the square of the fundamental frequency, ω^2, becomes

$$\omega^2 = f_s^{\,2} \frac{D}{\rho h a^4} \qquad (3.54)$$

Hence, $\omega = \frac{f_s}{a^2}\sqrt{\frac{D}{\rho h}}$ (3.55)

where, f_s is the coefficient of free vibration.

The Eqns (3.51) or (3.55) is the general expression for the fundamental natural frequency of a thin isotropic rectangular plate with aspect ratio, s = b/a.

Programs based on Eqns (3.53) and (3.55) are develop for free vibration analysis in section 3.5.

3.4 Continuous Plate

In this section of work, two types of continuous thin plates were analyzed, namely, continuous plate spanning in one way, and continuous plate spanning in two ways.

3.4.1 One-Way Continuous thin Plate

The one-way continuous thin plate is assumed to be simply supported at the outer edges and fixed along the internal edges. It is shown in Fig.3.1

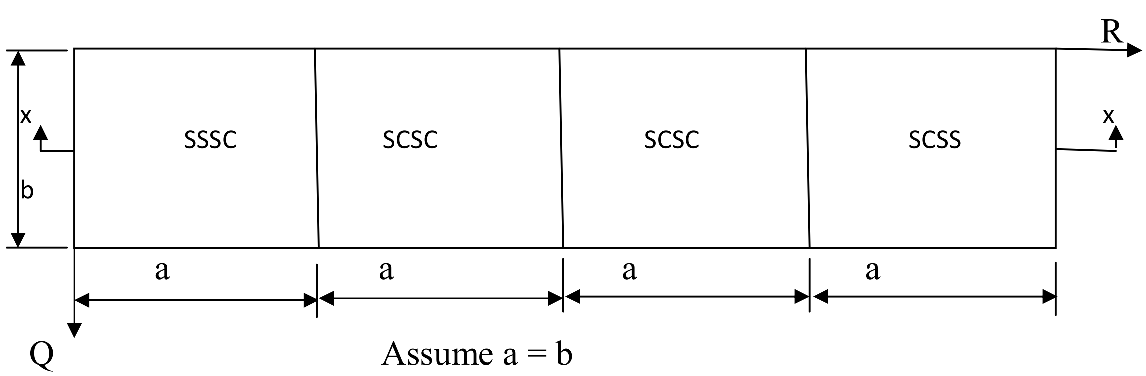


Fig 3.1: A Continuous plates spanning in one way

The continuous plate was divided into four panels of single rectangular plates, show in fig 3.1. Assuming the panels are symmetrical and have equal span length, a. The aspect ratio, (s= b/a), which is equal to unity, and the boundary conditions of each plate panel are substituted into the following version of Eqns (3.26) and (3.27);

FEM = $\beta_g q a^2$,

where g = R(x) or Q(y)

The expression is used to compute the values of the fixed edge or end moments (i.e FEMs) of each individual plate panel presented in Fig 3.2 and Table 4.23.

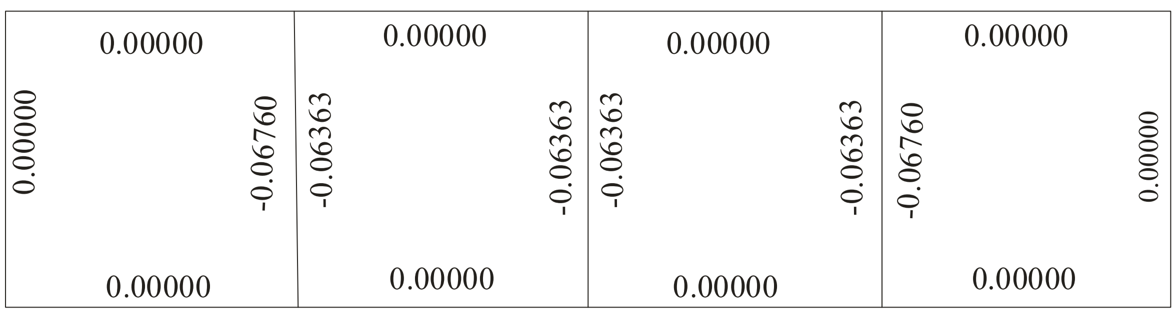


Fig. 3.2: One-way continuous plate showing fixed end moments of each panel.

Using beam analogy, a section x-x is taken through the center of each panel of the continuous plate as shown in Fig. 3.1, and represented as Fig. 3.3 for continuous plate in in one direction. It was analyzed by the use of stiffness method to obtain the support and span moments as follows:

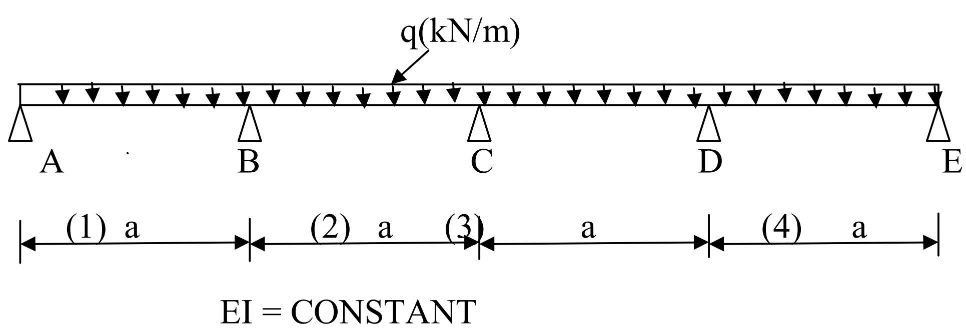


Fig 3.3a: Section x-x of the continuous plate in spanning one way

The rotations, Φ, at the supports due to the applied load are shown in Fig. 3.3b.

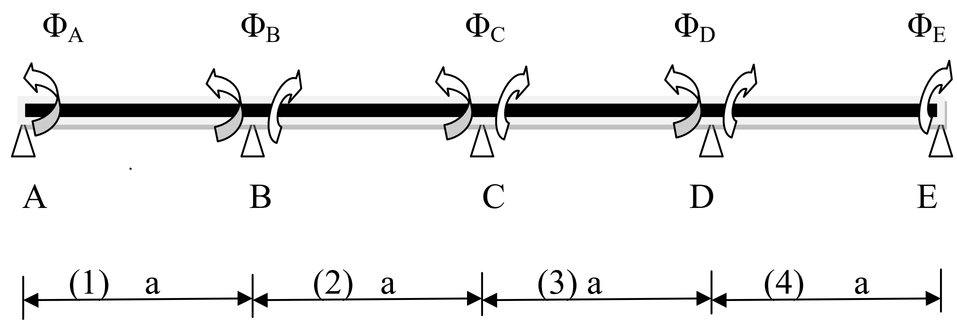


Fig 3.3b: Rotations at the supports of section x-x of the continuous beam.

The beam element stiffness, k_e, of the various spans (elements), are as presented below in a matrix form as follows:

For the first span (element), AB, having the length 'a', the stiffness, k_{e_1} is given by Eqn (3.56):

$$k_{e_1} = \begin{vmatrix} 4EI/L_1 & 2EI/L_1 \\ 2EI/L_1 & 4EI/L_1 \end{vmatrix} = EI/a \begin{vmatrix} 4 & 2 \\ 2 & 4 \end{vmatrix} \quad (3.56)$$

Similarly, for the second span (element), BC, having the length 'a', the stiffness, k_{e_2} is given as Eqn (3.57):

$$k_{e_2} = \begin{vmatrix} 4EI/L_2 & 2EI/L_2 \\ 2EI/L_2 & 4EI/L_2 \end{vmatrix} = EI/a \begin{vmatrix} 4 & 2 \\ 2 & 4 \end{vmatrix} \quad (3.57)$$

Also, for the third span (element), CD, having the length 'a', the stiffness, k_{e_3} is given by Eqn (3.58):

$$k_{e_3} = \begin{vmatrix} 4EI/L_3 & 2EI/L_3 \end{vmatrix} = EI/a \begin{vmatrix} 4 & 2 \end{vmatrix} \quad (3.58)$$

$$\begin{vmatrix} 2EI/L_3 & 4EI/L_3 \end{vmatrix} \qquad \begin{vmatrix} 2 & 4 \end{vmatrix}$$

For the fourth span (element), DE, having the length ‘a’, the stiffness, k_{e_4} is given by Eqn (3.59):

$$k_{e_4} = \begin{vmatrix} 4EI/L_4 & 2EI/L_4 \\ 2EI/L_4 & 4EI/L_4 \end{vmatrix} = EI/a \begin{vmatrix} 4 & 2 \\ 2 & 4 \end{vmatrix} \qquad (3.59)$$

Now, assemblage of the individual element stiffness matrices of Eqns (3.56), (3.57), (3.58), and (3.59) into a global or structural stiffness matrix, K, of the entire plate, yields Eqn (3.60)

$$K= \; EI/a \begin{vmatrix} 4 & 2 & 0 & 0 & 0 \\ 2 & 8 & 2 & 0 & 0 \\ 0 & 2 & 8 & 2 & 0 \\ 0 & 0 & 2 & 8 & 2 \\ 0 & 0 & 0 & 2 & 4 \end{vmatrix} \qquad (3.60)$$

From Fig. 3.2, the fixed end moments (FEM) for the various supports are as follows:

$FEM_{AB} = FEM_{ED} = -0kNm;$

$FEM_{BA} = FEM_{DE} = -0.067599qa^2kNm;$

FEM_{BC}=FEMCB=FEM_{CD}=FEM_{DC} = $-0.063626qa^2kNm;$

Arranging the FEMs in matrix form, yields Eqn (3.61)

$$\begin{vmatrix} FEM_{AB} \\ FEM_{BA} \\ FEM_{BC} \\ FEM_{CB} \end{vmatrix} = \begin{vmatrix} 0.00000\ qa^2 \\ -0.06760\ qa^2 \\ -0.06363\ qa^2 \\ -0.06363\ qa^2 \end{vmatrix} \qquad (3.61)$$

$$\begin{vmatrix} FEM_{CD} \\ FEM_{DC} \\ FEM_{DE} \\ FEM_{ED} \end{vmatrix} \quad \begin{vmatrix} -0.06363\ qa^2 \\ -0.06363\ qa^2 \\ -0.06760\ qa^2 \\ 0.00000\ qa^2 \end{vmatrix}$$

After factoring qa^2 out, Eqn (3.61) becomes

$$\begin{vmatrix} FEM_{AB} \\ FEM_{BA} \\ FEM_{BC} \\ FEM_{CB} \\ FEM_{CD} \\ FEM_{DC} \\ FEM_{DE} \\ FEM_{ED} \end{vmatrix} = qa^2 \begin{vmatrix} 0.00000 \\ -0.06760 \\ -0.06363 \\ -0.06363 \\ -0.06363 \\ -0.06363 \\ -0.06760 \\ 0.00000 \end{vmatrix} \qquad (3.62)$$

The final fixed end moment at each support, is given by Eqn (3.63)

$$\begin{vmatrix} FEM_{A} \\ FEM_{B} \\ FEM_{C} \\ FEM_{D} \\ FEM_{E} \end{vmatrix} = qa^2 \begin{vmatrix} 0 \\ -0.067599+0.063626 \\ -0.063626+0.063626 \\ -0.063626+0.067599 \\ 0 \end{vmatrix} = qa^2 \begin{vmatrix} 0.00000 \\ -0.00397 \\ 0.00000 \\ 0.00397 \\ 0.00000 \end{vmatrix} \qquad (3.63)$$

But, in stiffness method of analysis, applied load or reaction is proportional the displacement cause by that reaction. Hence,

$$F \alpha \Delta \qquad (3.64a)$$

$$F = k \Delta \qquad (3.64b)$$

where, Δ is displacement

Representing Eqn (3.64b) in a matrix form, and after introducing a proportionality constant, K (i.e stiffness), gives Eqn (3.65),

$$|FEM| = |K| \cdot |\Delta| \qquad (3.65)$$

Substituting Eqns (3.60) and (3.63) into Eqn (3.65), yields Eqn (3.66)

$$\begin{vmatrix} 0.00000 \\ -0.00397 \\ 0.00000 \\ 0.00397 \\ 0.00000 \end{vmatrix} = EI/a \begin{vmatrix} 4 & 2 & 0 & 0 & 0 \\ 2 & 8 & 2 & 0 & 0 \\ 0 & 2 & 8 & 2 & 0 \\ 0 & 0 & 2 & 8 & 2 \\ 0 & 0 & 0 & 2 & 4 \end{vmatrix} * \begin{vmatrix} \Phi_A \\ \Phi_B \\ \Phi_C \\ \Phi_D \\ \Phi_E \end{vmatrix} \quad (3.66)$$

where the unknown displacements, Δ is given by

$$\left| \Delta \right| = \begin{vmatrix} \Phi_A \\ \Phi_B \\ \Phi_C \\ \Phi_D \\ \Phi_E \end{vmatrix} \quad (3.67)$$

Transposing Eqn (3.66) and obtaining the inverse of the stiffness matrix, K, yields Eqn (3.68).

$$\begin{vmatrix} \Phi_A \\ \Phi_B \\ \Phi_C \\ \Phi_D \\ \Phi_E \end{vmatrix} = a/EI \begin{vmatrix} 0.28869 & -0.07738 & 0.020833 & -0.00595 & 0.002976 \\ -0.07738 & 0.154762 & -0.04167 & 0.011905 & -0.00595 \\ 0.020833 & -0.04167 & 0.145833 & -0.04167 & 0.020833 \\ -0.00595 & 0.011905 & -0.04167 & 0.154762 & -0.07738 \\ 0.002976 & -0.00595 & 0.020833 & -0.07738 & 0.28869 \end{vmatrix} * \begin{vmatrix} 0.00000 \\ -0.00397 \\ 0.00000 \\ 0.00397 \\ 0.00000 \end{vmatrix} qa^2 \quad (3.68)$$

Executing the matrix multiplication of the RHS of Eqn (3.68), gives the displacements at the supports as follows:

$$\begin{vmatrix} \Phi_A \\ \Phi_B \\ \Phi_C \\ \Phi_D \\ \Phi_E \end{vmatrix} = qa^3/EI \begin{vmatrix} 0.000284 \\ -0.00057 \\ 0.00000 \\ 0.000567 \\ -0.00028 \end{vmatrix} \quad (3.69)$$

To obtain the member or element forces, MF,substitute the stiffness of each element [i.e Eqns (3.56), (3.57), (3.58), and (3.59)] and member displacements of Eqn (3.69) into Eqn (3.70), which is a modified expression of Eqn (3.65) , and carrying out matrix multiplication, gives Eqns (3.71) to (3.74).

$$\begin{vmatrix} MF \end{vmatrix} = \begin{vmatrix} k_e \end{vmatrix} . \begin{vmatrix} \Phi_e \end{vmatrix} \qquad (3.70)$$

For member/element AB

$$\begin{vmatrix} MF_A \\ MF_B \end{vmatrix} = EI/a \begin{vmatrix} 4 & 2 \\ 2 & 4 \end{vmatrix} * qa^3/EI \begin{vmatrix} 0.000284 \\ -0.00057 \end{vmatrix} = qa^2 \begin{vmatrix} 0 \\ -0.0017 \end{vmatrix} \qquad (3.71)$$

For member/element BC

$$\begin{vmatrix} MF_B \\ MF_C \end{vmatrix} = EI/a \begin{vmatrix} 4 & 2 \\ 2 & 4 \end{vmatrix} * qa^3/EI \begin{vmatrix} -0.00057 \\ 0.00000 \end{vmatrix} = qa^2 \begin{vmatrix} -0.00227 \\ -0.00113 \end{vmatrix} \qquad (3.72)$$

For member/element CD

$$\begin{vmatrix} MF_C \\ MF_D \end{vmatrix} = EI/a \begin{vmatrix} 4 & 2 \\ 2 & 4 \end{vmatrix} * qa^3/EI \begin{vmatrix} -0.00000 \\ 0.000567 \end{vmatrix} = qa^2 \begin{vmatrix} 0.001134 \\ 0.002269 \end{vmatrix} \qquad (3.73)$$

For member/element DE

$$\begin{vmatrix} MF_D \\ MF_E \end{vmatrix} = EI/a \begin{vmatrix} 4 & 2 \\ 2 & 4 \end{vmatrix} * qa^3/EI \begin{vmatrix} 0.000567 \\ -0.00028 \end{vmatrix} = qa^2 \begin{vmatrix} 0.001701 \\ 0.00000 \end{vmatrix} \qquad (3.74)$$

To obtain the final support moments, M, at each support of the continuous plate, the values of the member or element end forces or reactions, MF, at each support of that element is subtracted from the fixed end moments at the supports given by Eqn (3.75).

$$\begin{vmatrix} M \end{vmatrix} = \begin{vmatrix} FEM \end{vmatrix} - \begin{vmatrix} MF \end{vmatrix} \qquad (3.75)$$

For member/element AB,

$$\begin{vmatrix} M_{AB} \\ M_{BA} \end{vmatrix} = qa^2 \begin{vmatrix} 0.0000 \\ -0.0676 \end{vmatrix} - qa^2 \begin{vmatrix} 0.0000 \\ -0.0017 \end{vmatrix} = qa^2 \begin{vmatrix} 0.0000 \\ -0.0659 \end{vmatrix} kNm \qquad (3.76)$$

For member/element BC,

$$\begin{vmatrix} M_{BC} \\ M_{CB} \end{vmatrix} = qa^2 \begin{vmatrix} -0.06363 \\ -0.06363 \end{vmatrix} - qa^2 \begin{vmatrix} -0.00227 \\ -0.00113 \end{vmatrix} = qa^2 \begin{vmatrix} -0.06136 \\ -0.06250 \end{vmatrix} \text{kNm} \qquad (3.77)$$

For member/element CD,

$$\begin{vmatrix} M_{CD} \\ M_{DC} \end{vmatrix} = qa^2 \begin{vmatrix} -0.06363 \\ -0.06363 \end{vmatrix} - qa^2 \begin{vmatrix} 0.001134 \\ 0.002269 \end{vmatrix} = qa^2 \begin{vmatrix} -0.06476 \\ -0.06590 \end{vmatrix} \text{kNm} \qquad (3.78)$$

For member/element DE,

$$\begin{vmatrix} M_{DE} \\ M_{ED} \end{vmatrix} = qa^2 \begin{vmatrix} -0.06760 \\ 0.00000 \end{vmatrix} - qa^2 \begin{vmatrix} -0.001701 \\ 0.00000 \end{vmatrix} = qa^2 \begin{vmatrix} -0.06930 \\ 0.00000 \end{vmatrix} \text{kNm} \qquad (3.79)$$

Therefore, the final support moments are as shown in Eqn (3.80) as follows:

$$\begin{vmatrix} M_{AB} \\ M_{BA} \\ M_{BC} \\ M_{CB} \\ M_{CD} \\ M_{DC} \\ M_{DE} \\ M_{ED} \end{vmatrix} = qa^2 \begin{vmatrix} 0.00000 \\ -0.06950 \\ -0.06136 \\ -0.06250 \\ -0.06476 \\ -0.06590 \\ -0.06930 \\ 0.00000 \end{vmatrix} \text{kNm} \qquad (3.80)$$

To obtain the span moment, M_{span}, we use the expression in Eqn (3.81), which is based on the assumption that each of the spans is consider simply supported. Then, the static span moment of each span minus the average of the two supports moments of that element/member.

$$M_{span} = 0.125qa^2 - 0.5(M_{YZ}+M_{ZY})qa^2 \qquad (3.81)$$

Where, y and z represent the supports of that span/element.

Therefore, substituting the values of the final support moments given by Eqn (3.80) into Eqn (3.81), yields the following results:

$M_{span(1)} = 0.0932qa^2kNm$ (3.81a)

$M_{span(2)} = 0.06137qa^2kNm$ (3.81b)

$M_{span(3)} = 0.05939qa^2kNm$ (3.81c)

$M_{span(4)} = 0.0912qa^2kNm$ (3.81d)

The discussions on these results are done in chapter four of this work. The bending moment diagram is plotted as shown in Fig. 3.4.

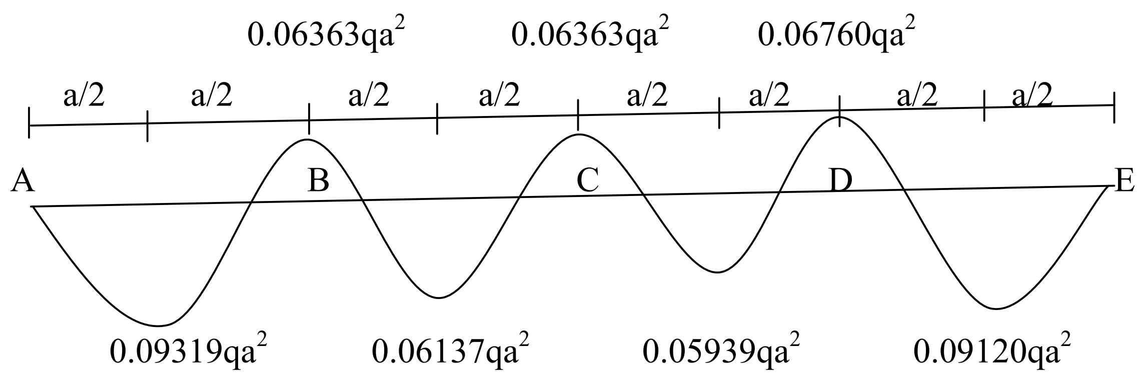


Fig.3.4: Section x-x BMD (kNm) for one-way continuous plate

3.4.2 Two-Way Continuous Plate

In this section, a continuous plate in two ways was analyzed in both x- and y-directions. In other to carry out the analysis with ease, the continuous plate was divided into twelve single panels in both ways forming three continuous strips in x- direction and four continuous strips in y- direction as shown in Fig. 3.5.

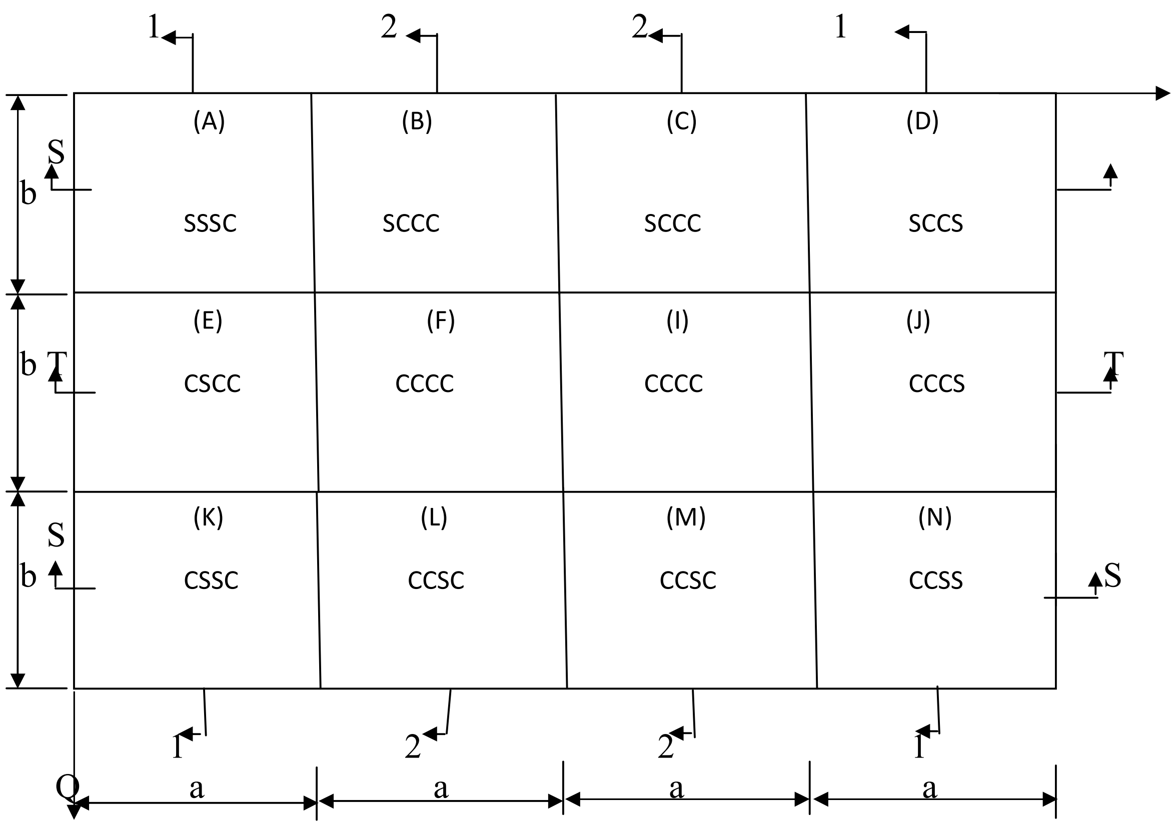


Fig. 3.5: Two-Way continuous plate

Considering the symmetry of the beam, with each single panel being a square plate having a dimension ‘a’ along x-direction, dimension ‘b’ along y- direction. The aspect ratio, b/a, is unity. Assumme the outer edges of the continuous plate to be simply supported and the internal edges to be fixed. The fixed edge moments of each single panel were obtained by substituting the boundary conditions of each individual plate into Eqns (3.26) and (3.27) hereby represented generally as, FEM = $\beta_g qa^2$, where g = R(x) - or Q(x) -direction.The values of the coefficients, β_g, of the fixed edge moments for the individual panels are presented in Fig. 3.6 and Tables 4.24– 4.27.

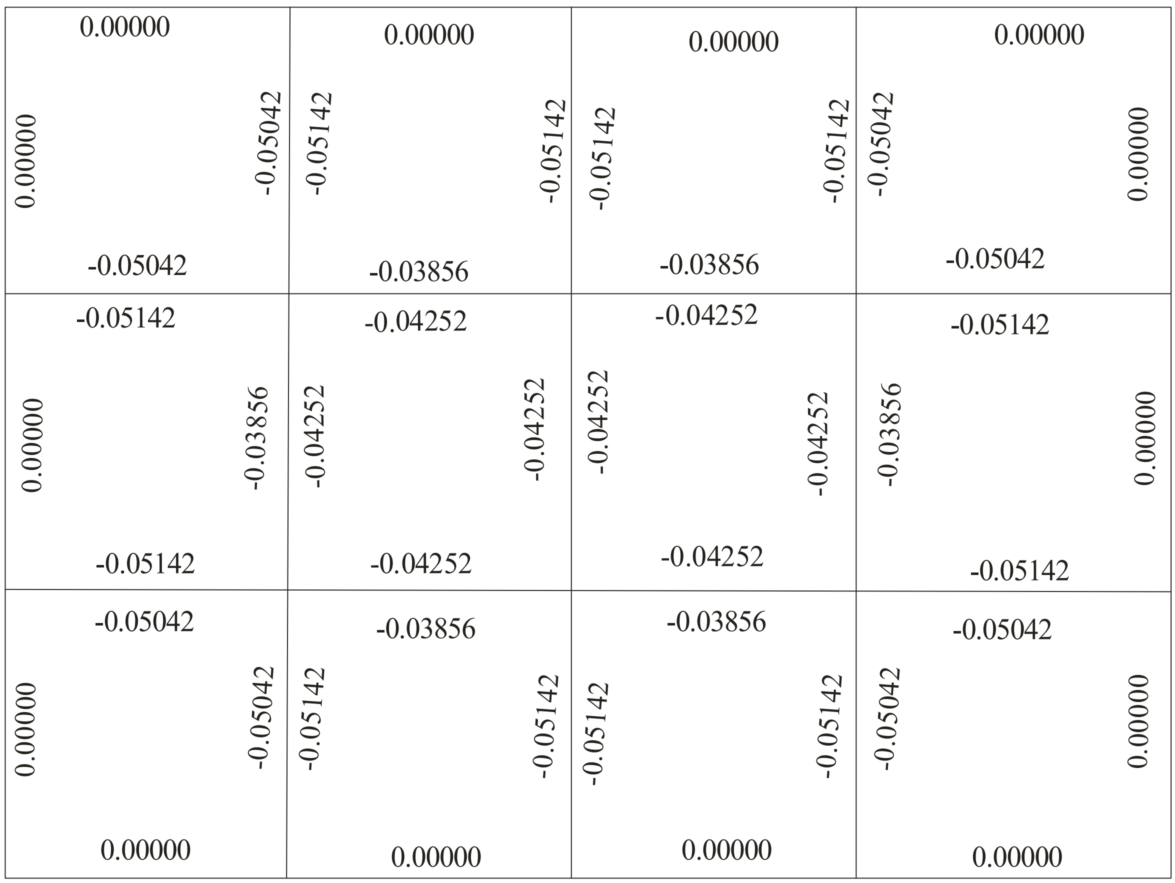
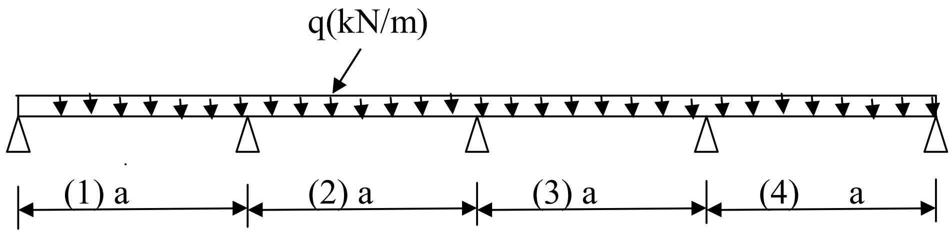


Fig.3.6: Fixed End Moments of a two- way continuous plate.

In a similar manner, beam analogy was used to analyze for the supports and span moment of the plate. In doing so, sections like: S-S, T-T, 1-1, and 2-2 were taken through the center of each strip as shown in fig. 3.5. Thereafter, element stiffness method is use in the analysis of the two-way continuous plate.

For the S-S section shown in fig. 3.7a, the support and span moments are analysed as follows:



EI = Constant

Fig 3.7a: Section S-S of the continuous plate.

The rotations at the supports due to the applied load are shown in fig. 3.7b.

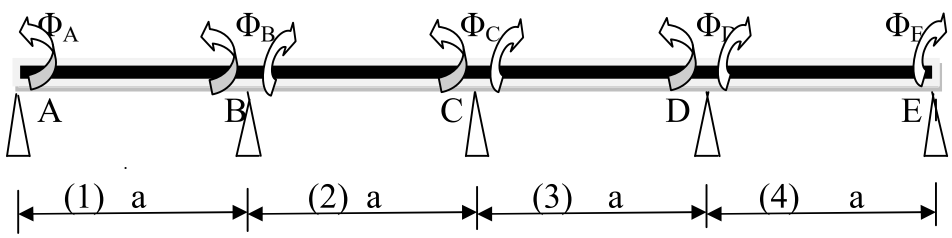


Fig 3.7b: Rotations at the supports section S –S of the continuous plate.

It was assume that the spans are equal, i.e, AB = BC =CD =DE and the aspect ratio, s = b/a = 1.

The beam element stiffness, k_e, of the various spans (elements) are as presented below in matrix form as follows:

For the first span (element), AB, having the length 'a', the stiffness, $k_{e1,}$ is given by Eqn (3.82):

$$k_{e1} = \begin{vmatrix} 4EI/L_1 & 2EI/L_1 \\ 2EI/L_1 & 4EI/L_1 \end{vmatrix} = EI/a \begin{vmatrix} 4 & 2 \\ 2 & 4 \end{vmatrix} \qquad (3.82)$$

Similarly, for the second span (element), BC, having the length 'a', the stiffness, $k_{e2,}$ is given by Eqn (3.83):

$$k_{e3} = \begin{vmatrix} 4EI/L_2 & 2EI/L_2 \\ 2EI/L_2 & 4EI/L_2 \end{vmatrix} = EI/a \begin{vmatrix} 4 & 2 \\ 2 & 4 \end{vmatrix} \quad (3.83)$$

Also, for the third span (element), CD, having the length 'a', the stiffness, $k_{e3,}$ is given by Eqn (3.84):

$$k_{e3} = \begin{vmatrix} 4EI/L_3 & 2EI/L_3 \\ 2EI/L_3 & 4EI/L_3 \end{vmatrix} = EI/a \begin{vmatrix} 4 & 2 \\ 2 & 4 \end{vmatrix} \quad (3.84)$$

For the forth span (element), DE, having the length 'a', the stiffness, $k_{e4,}$ is given by Eqn (3.85):

$$k_{e4} = \begin{vmatrix} 4EI/L_4 & 2EI/L_4 \\ 2EI/L_4 & 4EI/L_4 \end{vmatrix} = EI/a \begin{vmatrix} 4 & 2 \\ 2 & 4 \end{vmatrix} \quad (3.85)$$

Now, combining all the individual element stiffness matrices of Eqns (3.82)-(3.85) into a global or structural stiffness matrix, K, of the entire section of the plate, gives Eqn (3.86).

$$K = EI/a \begin{vmatrix} 4 & 2 & 0 & 0 & 0 \\ 2 & 8 & 2 & 0 & 0 \\ 0 & 2 & 8 & 2 & 0 \\ 0 & 0 & 2 & 8 & 2 \\ 0 & 0 & 0 & 2 & 4 \end{vmatrix} \quad (3.86)$$

From Fig. 3.7a, the fixed end moments, (FEM) for the various supports, are as follows:

$FEM_{AB} = FEM_{ED} = -0kNm;$

$FEM_{BA} = FEM_{DE} = -0.05042qa^2kNm;$

$FEM_{BC}=FEM_{CB}=FEM_{CD}=FEM_{DC} = -0.05142qa^2kNm;$

Representing the FEMs in matrix form, yields Eqn (3.87)

$$\begin{vmatrix} FEM_{AB} \\ FEM_{BA} \\ FEM_{BC} \\ FEM_{CB} \\ FEM_{CD} \\ FEM_{DC} \\ FEM_{DE} \\ FEM_{ED} \end{vmatrix} = \begin{vmatrix} 0.00000\ qa^2 \\ -0.05042\ qa^2 \\ -0.05142\ qa^2 \\ -0.05142\ qa^2 \\ -0.05142\ qa^2 \\ -0.05142\ qa^2 \\ -0.05042\ qa^2 \\ 0.00000\ qa^2 \end{vmatrix} \qquad (3.87)$$

And factoring qa^2 out, Eqn (3.87) becomes Eqn (3.88)

$$\begin{vmatrix} FEM_{AB} \\ FEM_{BA} \\ FEM_{BC} \\ FEM_{CB} \\ FEM_{CD} \\ FEM_{DC} \\ FEM_{DE} \\ FEM_{ED} \end{vmatrix} = qa^2 \begin{vmatrix} 0.00000 \\ -0.05042 \\ -0.05142 \\ -0.05142 \\ -0.05142 \\ -0.05142 \\ -0.05042 \\ 0.00000 \end{vmatrix} \qquad (3.88)$$

Thus, the final fixed end moment in each support, is obtained by summing up moment at each support.

$$\begin{vmatrix} FEM_{A} \\ FEM_{B} \\ FEM_{C} \\ FEM_{D} \\ FEM_{E} \end{vmatrix} = qa^2 \begin{vmatrix} 0 \\ -0.05042+0.05142 \\ -0.05142+0.05142 \\ -0.05142+0.05142 \\ 0 \end{vmatrix} = qa^2 \begin{vmatrix} 0.00000 \\ 0.001 \\ 0.00000 \\ -0.001 \\ 0.00000 \end{vmatrix} \qquad (3.89)$$

It will be recalled that, in stiffness method of analysis, applied reaction or load is proportional to the displacement caused by that reaction (See Eqn (3.65)

Substituting Eqns (3.86) and (3.89) into Eqn (3.65), yields Eqn (3.90)

$$\begin{vmatrix} 0.00000 \\ 0.001 \\ 0.00000 \\ -0.001 \\ 0.00000 \end{vmatrix} = EI/a \begin{vmatrix} 4 & 2 & 0 & 0 & 0 \\ 2 & 8 & 2 & 0 & 0 \\ 0 & 2 & 8 & 2 & 0 \\ 0 & 0 & 2 & 8 & 2 \\ 0 & 0 & 0 & 2 & 4 \end{vmatrix} * \begin{vmatrix} \Phi_A \\ \Phi_B \\ \Phi_C \\ \Phi_D \\ \Phi_E \end{vmatrix} \quad (3.90)$$

where the unknown displacements, Δ, is given by Eqn (3.91)

$$\begin{vmatrix} \Delta \end{vmatrix} = \begin{vmatrix} \Phi_A \\ \Phi_B \\ \Phi_C \\ \Phi_D \\ \Phi_E \end{vmatrix} \quad (3.91)$$

Transposing Eqn (3.90) in order to obtain the displacements, Δ, yields Eqn (3.92), after the inverse of the stiffness matrix, K, has been determined.

$$\begin{vmatrix} \Phi_A \\ \Phi_B \\ \Phi_C \\ \Phi_D \\ \Phi_E \end{vmatrix} = a/EI \begin{vmatrix} 0.28869 & -0.07738 & 0.020833 & -0.00595 & 0.002976 \\ -0.07738 & 0.154762 & -0.04167 & 0.011905 & -0.00595 \\ 0.020833 & -0.04167 & 0.145833 & -0.04167 & 0.020833 \\ -0.00595 & 0.011905 & -0.04167 & 0.154762 & -0.07738 \\ 0.002976 & -0.00595 & 0.020833 & -0.07738 & 0.28869 \end{vmatrix} * \begin{vmatrix} 0.00000 \\ 0.001 \\ 0.00000 \\ -0.001 \\ 0.00000 \end{vmatrix} qa^2 \quad (3.92)$$

Simplifying the Eqn (3.92), gives Eqn (3.93)

$$\begin{vmatrix} \Phi_A \\ \Phi_B \\ \Phi_C \\ \Phi_D \\ \Phi_E \end{vmatrix} = qa^3/EI \begin{vmatrix} -0.00007 \\ 0.000143 \\ 0.00000 \\ -0.00014 \\ 0.00007 \end{vmatrix} \quad (3.93)$$

To obtain the member or element reactions, MF, substitute respectively the stiffness of each element [i.e Eqns (3.82), (3.83), (3.84), and (3.85)], and the member displacement given by Eqn (3.93) into Eqn (3.70). These yields Eqns (3.94) -(3.97) respectively.

For member/element AB , the reactions, MF, are as follows:

$$\begin{vmatrix} MF_A \\ MF_B \end{vmatrix} = EI/a \begin{vmatrix} 4 & 2 \\ 2 & 4 \end{vmatrix} * qa^3/EI \begin{vmatrix} -0.00007 \\ 0.00014 \end{vmatrix} = qa^2 \begin{vmatrix} 0 \\ 0.00429 \end{vmatrix} \quad (3.94)$$

For member/element BC, the reactions, MF, are given by Eqn (3.95)

$$\begin{vmatrix} MF_B \\ MF_C \end{vmatrix} = EI/a \begin{vmatrix} 4 & 2 \\ 2 & 4 \end{vmatrix} * qa^3/EI \begin{vmatrix} 0.000143 \\ 0.00000 \end{vmatrix} = qa^2 \begin{vmatrix} 0.000571 \\ 0.000286 \end{vmatrix} \quad (3.95)$$

For member/element CD

$$\begin{vmatrix} MF_C \\ MF_D \end{vmatrix} = EI/a \begin{vmatrix} 4 & 2 \\ 2 & 4 \end{vmatrix} * qa^3/EI \begin{vmatrix} 0.00000 \\ -0.00014 \end{vmatrix} = qa^2 \begin{vmatrix} -0.00029 \\ -0.0057 \end{vmatrix} \quad (3.96)$$

For member/element DE

$$\begin{vmatrix} MF_D \\ MF_E \end{vmatrix} = EI/a \begin{vmatrix} 4 & 2 \\ 2 & 4 \end{vmatrix} * qa^3/EI \begin{vmatrix} -0.00014 \\ 0.00007 \end{vmatrix} = qa^2 \begin{vmatrix} -0.00043 \\ 0.00000 \end{vmatrix} \quad (3.97)$$

To obtain the final support moments, M, at each support, the values of the member or element forces or reactions, MF, at each support of that element are subtracted from the fixed end moments at the supports in question using Eqn (3.75).

For member/element AB, the final support moment is obtained using Eqn (3.98)

$$\begin{vmatrix} M_{AB} \\ M_{BA} \end{vmatrix} = qa^2 \begin{vmatrix} 0 \\ -0.05 \end{vmatrix} - qa^2 \begin{vmatrix} 0 \\ 0.0004 \end{vmatrix} = qa^2 \begin{vmatrix} 0 \\ -0.05085 \end{vmatrix} \text{ kNm} \quad (3.98)$$

For member/element BC, the final support moment is calculated as follows

$$\begin{vmatrix} M_{BC} \\ M_{CB} \end{vmatrix} = qa^2 \begin{vmatrix} -0.05142 \\ -0.05142 \end{vmatrix} - qa^2 \begin{vmatrix} 0.000571 \\ 0.000286 \end{vmatrix} = qa^2 \begin{vmatrix} -0.05199 \\ -0.05171 \end{vmatrix} \text{ kNm} \quad (3.99)$$

For member/Element CD,

$$\begin{vmatrix} M_{CD} \\ M_{DC} \end{vmatrix} = qa^2 \begin{vmatrix} -0.05142 \\ -0.05142 \end{vmatrix} - qa^2 \begin{vmatrix} -0.00029 \\ -0.00057 \end{vmatrix} = qa^2 \begin{vmatrix} -0.05113 \\ -0.05085 \end{vmatrix} \text{kNm} \qquad (3.100)$$

For member/Element DE,

$$\begin{vmatrix} M_{DE} \\ M_{ED} \end{vmatrix} = qa^2 \begin{vmatrix} -0.05042 \\ -0.00000 \end{vmatrix} - qa^2 \begin{vmatrix} -0.00043 \\ 0.00000 \end{vmatrix} = qa^2 \begin{vmatrix} -0.04999 \\ 0.00000 \end{vmatrix} \text{kNm} \qquad (3.101)$$

Therefore, the final support moments are as follows:

$$\begin{vmatrix} M_{AB} \\ M_{BA} \\ M_{BC} \\ M_{CB} \\ M_{CD} \\ M_{DC} \\ M_{DE} \\ M_{ED} \end{vmatrix} = qa^2 \begin{vmatrix} 0.00000 \\ -0.05085 \\ -0.05199 \\ -0.05171 \\ -0.05113 \\ -0.05085 \\ -0.04999 \\ 0.00000 \end{vmatrix} \text{kNm} \qquad (3.102)$$

To obtain the span moment, M_{span}, use the expression Eqn (3.81), which is based on the assumption that each of the spans, is consider simply supported. The span moment, Mspan, is equal to the static span moment of each span minus the average of the two supports moments of that element/member. Thus,

$$M_{span} = 0.125qa^2 - 0.5(M_{YZ}+M_{ZY})qa^2 \text{ (3.81)} \qquad (3.103)$$

Substituting the values of the final support moments in Eqn (3.102) into Eqn (3.81), yields the following results:

$$M_{span(1)} = 0.09929qa^2 \text{ kNm} \qquad (3.104)$$

$$M_{span(2)} = 0.07358qa^2 \text{ kNm} \qquad (3.105)$$

$$M_{span(3)} = 0.07408qa^2 \text{ kNm} \quad (3.106)$$

$$M_{span(4)} = 0.09979qa^2 \text{kNm} \quad (3.107)$$

The bending moment diagram is plotted as shown in Fig. 3.8.

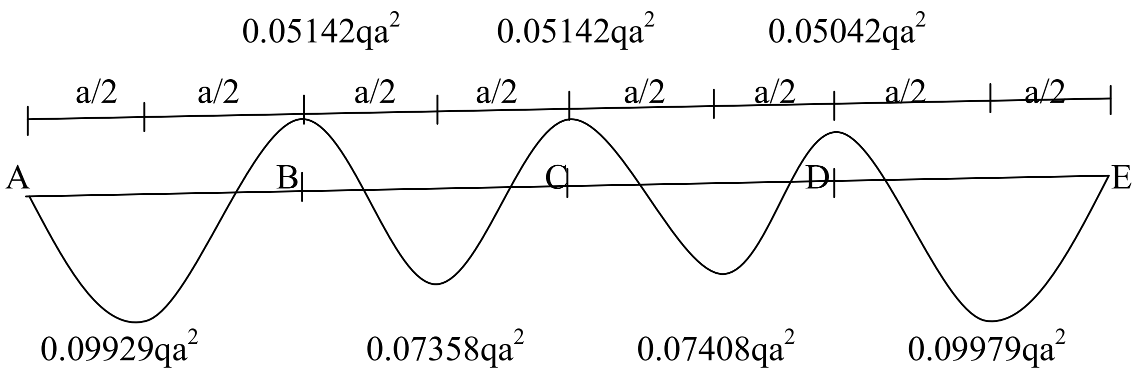


Fig.3.8: Bending Moment Diagram at Section S-S BMD of a two-way continuous plate

For Section T-T of the two-way continuous plate, the following results or values, were obtained using the same procedure:

Fixed end moments at the ends of the elements are given by Eqn (3.108)

$$\begin{vmatrix} FEM_{AB} \\ FEM_{BA} \\ FEM_{BC} \\ FEM_{CB} \\ FEM_{CD} \\ FEM_{DC} \\ FEM_{DE} \\ FEM_{ED} \end{vmatrix} = qa^2 \begin{vmatrix} 0.00000 \\ -0.03856 \\ -0.04252 \\ -0.04252 \\ -0.04252 \\ -0.04252 \\ -0.03856 \\ 0.00000 \end{vmatrix} \quad (3.108)$$

The final fixed end moment in each support is given Eqn (3.109) as

$$\begin{vmatrix} FEM_A \\ FEM_B \\ FEM_C \\ FEM_D \\ FEM_E \end{vmatrix} = qa^2 \begin{vmatrix} 0 \\ -0.03856+0.04252 \\ -0.04252+0.04252 \\ -0.04252+0.03856 \\ 0 \end{vmatrix} = qa^2 \begin{vmatrix} 0.00000 \\ 0.00396 \\ 0.00000 \\ -0.00396 \\ 0.00000 \end{vmatrix} \qquad (3.109)$$

The displacements at the supports are given by Eqn (3.110):

$$\begin{vmatrix} \Phi_A \\ \Phi_B \\ \Phi_C \\ \Phi_D \\ \Phi_E \end{vmatrix} = qa^3/EI \begin{vmatrix} 0.00028 \\ 0.000566 \\ 0.00000 \\ 0.00057 \\ -0.00028 \end{vmatrix} \qquad (3.110)$$

The member or element reactions or forces, MF, are calculated as follows:

For member/element AB, the end reactions are given by Eqn (3.111)

$$\begin{vmatrix} MF_A \\ MF_B \end{vmatrix} = EI/a \begin{vmatrix} 4 & 2 \\ 2 & 4 \end{vmatrix} * \quad qa^3/EI \begin{vmatrix} 0.00028 \\ 0.000566 \end{vmatrix} = qa^2 \begin{vmatrix} 0.0000 \\ 0.0017 \end{vmatrix} \qquad (3.111)$$

For member/element BC, the reactions at the ends, are as follows:

$$\begin{vmatrix} MF_B \\ MF_C \end{vmatrix} = EI/a \begin{vmatrix} 4 & 2 \\ 2 & 4 \end{vmatrix} * \quad qa^3/EI \begin{vmatrix} 0.000566 \\ 0.00000 \end{vmatrix} = qa^2 \begin{vmatrix} 0.00226 \\ 0.00113 \end{vmatrix} \qquad (3.112)$$

For member/element CD, member end forces can be determined using Eqn (3.113)

$$\begin{vmatrix} MF_C \\ MF_D \end{vmatrix} = EI/a \begin{vmatrix} 4 & 2 \\ 2 & 4 \end{vmatrix} * \quad qa^3/EI \begin{vmatrix} 0.00000 \\ -0.000567 \end{vmatrix} = qa^2 \begin{vmatrix} -0.00113 \\ -0.00227 \end{vmatrix} \qquad (3.113)$$

For member/element DE,

$$\begin{vmatrix} MF_D \\ MF_E \end{vmatrix} = EI/a \begin{vmatrix} 4 & 2 \\ 2 & 4 \end{vmatrix} * \quad qa^3/EI \begin{vmatrix} 0.000567 \\ -0.00028 \end{vmatrix} = qa^2 \begin{vmatrix} -0.00170 \\ 0.00000 \end{vmatrix} \qquad (3.114)$$

The final support moments at each support are obtained as follows:

For member/element AB, the final support is obtained using Eqn (3.115)

$$\begin{vmatrix} M_{AB} \\ M_{BA} \end{vmatrix} = qa^2 \begin{vmatrix} 0.00000 \\ -0.03856 \end{vmatrix} - qa^2 \begin{vmatrix} 0.00000 \\ 0.0017 \end{vmatrix} = qa^2 \begin{vmatrix} 0.00000 \\ -0.04026 \end{vmatrix} \text{kNm} \qquad (3.115)$$

For member/element BC, the final support moment is calculated as follows

$$\begin{vmatrix} M_{BC} \\ M_{CB} \end{vmatrix} = qa^2 \begin{vmatrix} -0.04252 \\ -0.04252 \end{vmatrix} - qa^2 \begin{vmatrix} 0.00226 \\ 0.00113 \end{vmatrix} = qa^2 \begin{vmatrix} -0.04478 \\ -0.04365 \end{vmatrix} \text{kNm} \qquad (3.116)$$

For member/Element CD,

$$\begin{vmatrix} M_{CD} \\ M_{DC} \end{vmatrix} = qa^2 \begin{vmatrix} -0.04252 \\ -0.04252 \end{vmatrix} - qa^2 \begin{vmatrix} -0.001134 \\ -0.002269 \end{vmatrix} = qa^2 \begin{vmatrix} -0.04139 \\ -0.04024 \end{vmatrix} \text{kNm} \qquad (3.117)$$

For member/Element DE,

$$\begin{vmatrix} M_{DE} \\ M_{ED} \end{vmatrix} = qa^2 \begin{vmatrix} -0.03856 \\ 0.00000 \end{vmatrix} - qa^2 \begin{vmatrix} -0.00170 \\ 0.00000 \end{vmatrix} = qa^2 \begin{vmatrix} -0.03686 \\ 0.00000 \end{vmatrix} \text{kNm} \qquad (3.118)$$

The final support moments are given by Eqn (3.119) as follows:

$$\begin{vmatrix} M_{AB} \\ M_{BA} \\ M_{BC} \\ M_{CB} \\ M_{CD} \\ M_{DC} \end{vmatrix} = qa^2 \begin{vmatrix} 0.00000 \\ -0.04026 \\ -0.04478 \\ -0.04365 \\ -0.04139 \\ -0.04024 \end{vmatrix} \text{kNm} \qquad (3.119)$$

M_{DE}		-0.03686
M_{ED}		0.00000

The span moments, M_{span}, are given as follows:

$$M_{span(1)} = 0.10374qa^2 \text{ kNm} \quad (3.120)$$

$$M_{span(2)} = 0.08248qa^2 \text{ kNm} \quad (3.121)$$

$$M_{span(3)} = 0.08447qa^2 \text{ kNm} \quad (3.122)$$

$$M_{span(4)} = 0.10573qa^2 \text{kNm} \quad (3.123)$$

The bending moment diagram is plotted as shown in fig.3.9.

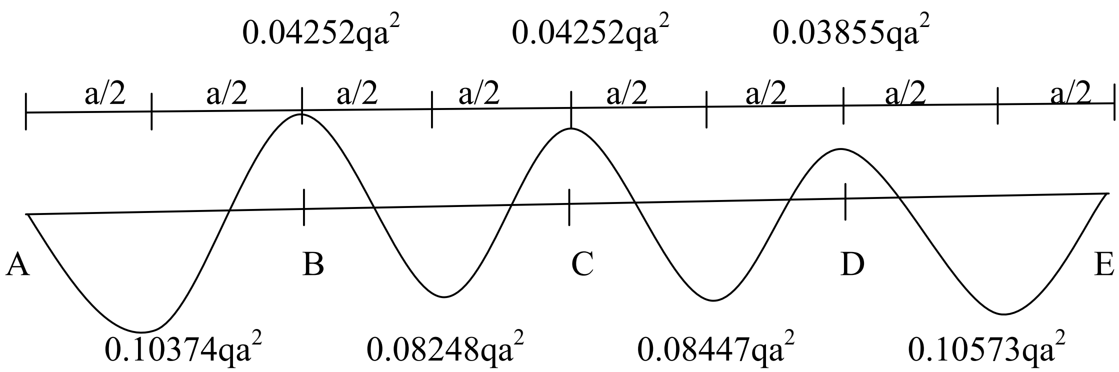


Fig.3.9: Bending Moment Diagram of Section T-T of two- way continuous plate

Considering the y-direction of the continuous plate, the first strip, section 1-1 is represented below:

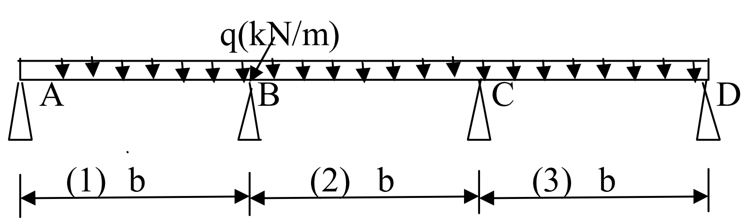


Fig 3.10a: Section 1-1 of the continuous plate

The rotations at the support due to applied load are shown in fig. 3.10b

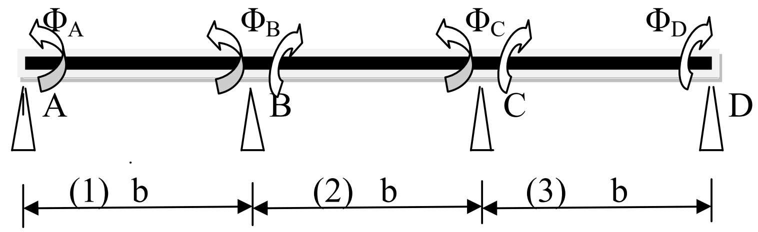


Fig 3.10b: Rotations at the supports of Section 1-1 of the continuous plate

Assume the spans are equal i.e AB = BC =CD =DE and the aspect ratio, s = b/a = 1.

The beam element stiffness, k_e of the various spans (elements) are as presented in the matrix form as follows:

For the first span (element), AB, having the length 'a', the stiffness, k_{e1} is given by Eqn (3.124):

$$k_{e1} = \begin{vmatrix} 4EI/L_1 & 2EI/L_1 \\ 2EI/L_1 & 4EI/L_1 \end{vmatrix} = EI/b \begin{vmatrix} 4 & 2 \\ 2 & 4 \end{vmatrix} \quad (3.124)$$

In a similarly way, for the second span (element), BC, having the length 'a', the stiffness, $k_{e2,}$ is given as Eqn (3.125):

$$k_{e2} = \begin{vmatrix} 4EI/L_2 & 2EI/L_2 \\ 2EI/L_2 & 4EI/L_2 \end{vmatrix} = EI/b \begin{vmatrix} 4 & 2 \\ 2 & 4 \end{vmatrix} \quad (3.125)$$

Also, for the third span (element), CD, with length 'a', the stiffness, $k_{e3,}$ is given by Eqn (3.126):

$$k_{e3} = \begin{vmatrix} 4EI/L_3 & 2EI/L_3 \\ 2EI/L_3 & 4EI/L_3 \end{vmatrix} = EI/b \begin{vmatrix} 4 & 2 \\ 2 & 4 \end{vmatrix} \quad (3.126)$$

Now, assembling the individual element stiffness matrices of Eqns (3.124) - (3.126), into a global or structural stiffness matrix, K, of the entire section of the plate, yields Eqn (3.127)

$$K = EI/b \begin{vmatrix} 4 & 2 & 0 & 0 \\ 2 & 8 & 2 & 0 \\ 0 & 2 & 8 & 2 \\ 0 & 0 & 2 & 4 \end{vmatrix} \quad (3.127)$$

From Fig. 3.10a, the fixed end moments, (FEM) for the various supports, are as follows:

$FEM_{AB} = FEM_{CD}$ = -0kNm;

$FEM_{BA} = FEM_{CD}$ = -0.05042qb^2kNm;

$FEM_{BC} = FEM_{CB}$= -0.05142qb^2kNm;

Representing the FEMs in a matrix form, yields Eqn (3.128)

$$\begin{vmatrix} FEM_{AB} \\ FEM_{BA} \\ FEM_{BC} \\ FEM_{CB} \\ FEM_{CD} \\ FEM_{DC} \end{vmatrix} = \begin{vmatrix} 0.00000\ qb^2 \\ -0.05042\ qb^2 \\ -0.05142\ qb^2 \\ -0.05142\ qb^2 \\ -0.05042\ qb^2 \\ 0.00000\ qb^2 \end{vmatrix} \quad (3.128)$$

And factoring qb^2 out, Eqn (3.128) becomes Eqn (3.129)

$$\begin{vmatrix} FEM_{AB} \\ FEM_{BA} \\ FEM_{BC} \\ FEM_{CB} \\ FEM_{CD} \\ FEM_{DC} \end{vmatrix} = qb^2 \begin{vmatrix} 0.00000 \\ -0.05042 \\ -0.05142 \\ -0.05142 \\ -0.05042 \\ 0.00000 \end{vmatrix} \quad (3.129)$$

The final fixed end moment in each support, obtained by summing up moment at each support, is given by Eqn (3.130)

$$\begin{vmatrix} FEM_A \\ FEM_B \\ FEM_C \\ FEM_D \end{vmatrix} = qb^2 \begin{vmatrix} 0 \\ -0.05042+0.05142 \\ -0.05142+0.05042 \\ 0 \end{vmatrix} = qb^2 \begin{vmatrix} 0.000 \\ 0.001 \\ -0.001 \\ 0.000 \end{vmatrix} \qquad (3.130)$$

It will be recalled that, in stiffness method of analysis, applied reaction or load is proportional the displacement caused by that reaction (See Eqn 3.65)

Substituting Eqns (3.127) and (3.131) into Eqn (3.65), yields Eqn (3.131)

$$\begin{vmatrix} 0.000 \\ 0.001 \\ -0.001 \\ 0.000 \end{vmatrix} = EI/b \begin{vmatrix} 4 & 2 & 0 & 0 \\ 2 & 8 & 2 & 0 \\ 0 & 2 & 8 & 2 \\ 0 & 0 & 2 & 4 \end{vmatrix} * \begin{vmatrix} \Phi_A \\ \Phi_B \\ \Phi_C \\ \Phi_D \end{vmatrix} \qquad (3.131)$$

where the unknown displacements, Δ is given by Eqn (3.132)

$$\begin{vmatrix} \Delta \end{vmatrix} = \begin{vmatrix} \Phi_A \\ \Phi_B \\ \Phi_C \\ \Phi_D \end{vmatrix} \qquad (3.132)$$

Transposing Eqn (3.131), in order to obtain the displacement, Δ, yields Eqn (3.133) after the inverse of the stiffness matrix, K, has been determined.

$$\begin{vmatrix} \Phi_A \\ \Phi_B \\ \Phi_C \\ \Phi_D \end{vmatrix} = b/EI \begin{vmatrix} 0.288889 & -0.07778 & 0.022222 & -0.01111 \\ -0.07778 & 0.155556 & -0.04444 & 0.022222 \\ 0.022222 & -0.04444 & 0.155556 & -0.07778 \\ -0.01111 & 0.022222 & -0.07778 & 0.288889 \end{vmatrix} * \begin{vmatrix} 0.00000 \\ 0.001 \\ 0.00000 \\ -0.001 \end{vmatrix} qb^2 \qquad (3.133)$$

Simplifying the Eqn (3.133), gives Eqn (3.134)

$$\begin{vmatrix} \Phi_A \\ \Phi_B \\ \Phi_C \\ \Phi_D \end{vmatrix} = qb^3/EI \begin{vmatrix} -0.0001 \\ 0.0002 \\ -0.0002 \\ 0.0001 \end{vmatrix} \qquad (3.134)$$

To obtain the member or element reactions, MF, substitute respectively the stiffness of each element [i.e Eqns (3.124), (3.125), and (3.126)], and the member displacement given by Eqn (3.134) into Eqn (3.65), gives Eqns (3.135) - (3.138), and obtaining the inverse of the stiffness matrix, K,

For member/element AB, the reactions, MF, are given by Eqn (3.135)

$$\begin{vmatrix} MF_A \\ MF_B \end{vmatrix} = EI/b \begin{vmatrix} 4 & 2 \\ 2 & 4 \end{vmatrix} * qb^3/EI \begin{vmatrix} -0.0001 \\ 0.0002 \end{vmatrix} = qb^2 \begin{vmatrix} 0.0000 \\ 0.0006 \end{vmatrix} \qquad (3.135)$$

For member/Element BC , the reactions are as follows:

$$\begin{vmatrix} MF_B \\ MF_C \end{vmatrix} = EI/b \begin{vmatrix} 4 & 2 \\ 2 & 4 \end{vmatrix} * qb^3/EI \begin{vmatrix} 0.0002 \\ -0.0002 \end{vmatrix} = qb^2 \begin{vmatrix} 0.0004 \\ -0.0004 \end{vmatrix} \qquad (3.136)$$

For member/Element CD

$$\begin{vmatrix} MF_C \\ MF_D \end{vmatrix} = EI/b \begin{vmatrix} 4 & 2 \\ 2 & 4 \end{vmatrix} * qb^3/EI \begin{vmatrix} -0.0002 \\ 0.0000 \end{vmatrix} = qb^2 \begin{vmatrix} -0.0006 \\ 0.0000 \end{vmatrix} \qquad (3.137)$$

To obtain the final support moments at each support, the values of the member or element forces or reactions, MF, at each support of that element are subtracted from the fixed end moments at the supports in question.

For member/element AB, the final support moments are obtain by as in Eqn (3.138)

$$\begin{vmatrix} M_{AB} \\ M_{BA} \end{vmatrix} = qb^2 \begin{vmatrix} 0.0000 \\ -0.05042 \end{vmatrix} - qb^2 \begin{vmatrix} 0.0000 \\ 0.0006 \end{vmatrix} = qb^2 \begin{vmatrix} 0 \\ -0.05102 \end{vmatrix} \text{kNm} \qquad (3.138)$$

For member/element BC, the final support moments are asp follows:

$$\begin{vmatrix} M_{BC} \\ M_{CB} \end{vmatrix} = qb^2 \begin{vmatrix} -0.05142 \\ -0.05142 \end{vmatrix} - qb^2 \begin{vmatrix} 0.0004 \\ -0.0004 \end{vmatrix} = qb^2 \begin{vmatrix} -0.05182 \\ -0.05102 \end{vmatrix} \text{kNm} \qquad (3.139)$$

For member/Element CD,

$$\begin{vmatrix} M_{AB} \\ M_{BA} \end{vmatrix} = qb^2 \begin{vmatrix} -0.05042 \\ 0.0000 \end{vmatrix} - qb^2 \begin{vmatrix} -0.0006 \\ 0.0000 \end{vmatrix} = qb^2 \begin{vmatrix} -0.04982 \\ 0.00000 \end{vmatrix} \text{kNm} \qquad (3.140)$$

Therefore, the final support moments are as follows:

$$\begin{vmatrix} M_{AB} \\ M_{BA} \\ M_{BC} \\ M_{CB} \\ M_{CD} \\ M_{DC} \end{vmatrix} = qb^2 \begin{vmatrix} 0.00000 \\ -0.05102 \\ -0.05182 \\ -0.05102 \\ -0.04982 \\ 0.00000 \end{vmatrix} \text{kNm} \qquad (3.141)$$

To obtain the span moment, M_{span}, use the expression Eqn (3.142), which is based on the assumption that each of the spans, is consider simply supported. The span moment, M_{span} is equal to the static span moment of each span minus the average of the two supports moments of that element/member. Thus

$$M_{span} = 0.125qb^2 - 0.5(M_{YZ}+M_{ZY})qb^2 \qquad (3.142)$$

Substituting the values of the final support moments in Eqn (3.141) into Eqn (3.142), yields the following results:

$$M_{span(1)} = 0.099299qb^2 \text{ kNm} \qquad (3.143)$$

$$M_{span(2)} = 0.07408qb^2 \text{ kNm} \qquad (3.144)$$

$$M_{span(3)} = 0.09979qb^2 \text{ kNm} \qquad (3.145)$$

The bending moment diagram is plotted as shown in fig. 3.11.

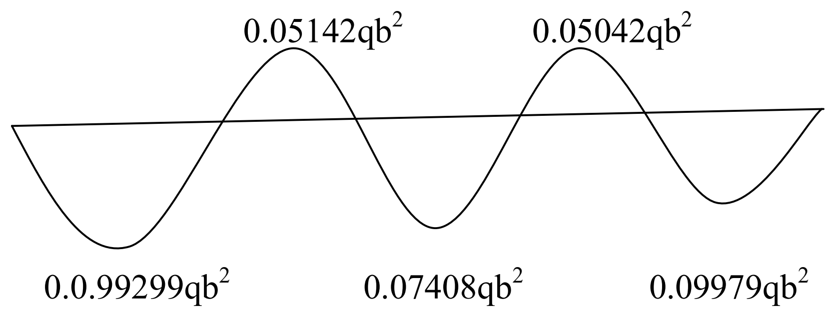


Fig.3.11: Bending Moment Digram at Section 1-1of two-way continuous plate

Following the same approach for section 2-2, the following results were obtained:

Fixed end moments at the ends of the elements are given by Eqn (3.146)

$$\begin{vmatrix} FEM_{AB} \\ FEM_{BA} \\ FEM_{BC} \\ FEM_{CB} \\ FEM_{CD} \\ FEM_{DC} \end{vmatrix} = qb^2 \begin{vmatrix} 0.00000 \\ -0.03856 \\ -0.04252 \\ -0.04252 \\ -0.03856 \\ 0.00000 \end{vmatrix} \qquad (3.146)$$

The final fixed end moment in each support is given Eqn (3.147) are

$$\begin{vmatrix} FEM_{A} \\ FEM_{B} \\ FEM_{C} \\ FEM_{D} \end{vmatrix} = qb^2 \begin{vmatrix} 0 \\ -0.03856+0.04252 \\ -0.04252+0.03856 \\ 0 \end{vmatrix} = qb^2 \begin{vmatrix} 0.00000 \\ 0.00396 \\ -0.00396 \\ 0.00000 \end{vmatrix} \qquad (3.147)$$

The displacements at the supports are given by Eqn (3.148):

$$\begin{vmatrix} \Phi_A \\ \Phi_B \\ \Phi_C \\ \Phi_D \end{vmatrix} = qb^3/EI \begin{vmatrix} -0.0004 \\ 0.000792 \\ -0.00079 \\ 0.000396 \end{vmatrix} \quad (3.148)$$

The members or elements reactions or forces, MF, are:

For member/element AB, the member forces are obtain as follows:

$$\begin{vmatrix} MF_A \\ MF_B \end{vmatrix} = EI/b \begin{vmatrix} 4 & 2 \\ 2 & 4 \end{vmatrix} * \quad qb^3/EI \begin{vmatrix} -0.0004 \\ 0.000792 \end{vmatrix} = qb^2 \begin{vmatrix} 0.00000 \\ 0.00238 \end{vmatrix} \quad (3.149)$$

For member/Element BC

$$\begin{vmatrix} MF_B \\ MF_C \end{vmatrix} = EI/b \begin{vmatrix} 4 & 2 \\ 2 & 4 \end{vmatrix} * \quad qb^3/EI \begin{vmatrix} 0.000792 \\ -0.00079 \end{vmatrix} = qb^2 \begin{vmatrix} 0.001584 \\ -0.00158 \end{vmatrix} \quad (3.150)$$

For member/Element CD

$$\begin{vmatrix} MF_C \\ MF_D \end{vmatrix} = EI/b \begin{vmatrix} 4 & 2 \\ 2 & 4 \end{vmatrix} * \quad qb^3/EI \begin{vmatrix} -0.00079 \\ 0.000396 \end{vmatrix} = qb^2 \begin{vmatrix} -0.00238 \\ 0.00000 \end{vmatrix} \quad (3.151)$$

The final support moments at each support are as follows:

For member/element AB, the final support moments are obtained as follows:

$$\begin{vmatrix} M_{AB} \\ M_{BA} \end{vmatrix} = qb^2 \begin{vmatrix} 0.0000 \\ -0.03856 \end{vmatrix} - qb^2 \begin{vmatrix} 0.0000 \\ 0.00238 \end{vmatrix} = qb^2 \begin{vmatrix} 0 \\ -0.04094 \end{vmatrix} \text{ kNm} \quad (3.152)$$

For member/element BC, the final support moments are given by Eqn (3.153)

$$\begin{vmatrix} M_{BC} \\ M_{CB} \end{vmatrix} = qb^2 \begin{vmatrix} -0.04252 \\ -0.04252 \end{vmatrix} - qb^2 \begin{vmatrix} 0.001584 \\ -0.00158 \end{vmatrix} = qb^2 \begin{vmatrix} -0.04410 \\ -0.04094 \end{vmatrix} \text{ kNm} \quad (3.153)$$

For member/Element CD,

$$\begin{vmatrix} M_{CD} \\ M_{DC} \end{vmatrix} = qb^2 \begin{vmatrix} -0.03856 \\ 0.00000 \end{vmatrix} - qb^2 \begin{vmatrix} -0.00238 \\ 0.00000 \end{vmatrix} = qb^2 \begin{vmatrix} -0.03618 \\ 0.00000 \end{vmatrix} \text{ kNm} \quad (3.154)$$

Therefore, the final support moments, are as given by Eqn (3.155):

$$\begin{vmatrix} M_{AB} \\ M_{BA} \\ M_{BC} \\ M_{CB} \\ M_{CD} \\ M_{DC} \end{vmatrix} = qb^2 \begin{vmatrix} 0.00000 \\ -0.04094 \\ -0.04410 \\ -0.04094 \\ -0.03618 \\ 0.00000 \end{vmatrix} \text{kNm} \qquad (3.155)$$

The span moments, M_{span}, are as follows:

$$M_{span(1)} = 0.10374qb^2 \text{ kNm} \qquad (3.156)$$

$$M_{span(2)} = 0.082446qb^2 \text{ kNm} \qquad (3.157)$$

$$M_{span(3)} = 0.10572qb^2 \text{ kNm} \qquad (3.158)$$

The bending moment diagram is plotted as shown in fig.3.13.

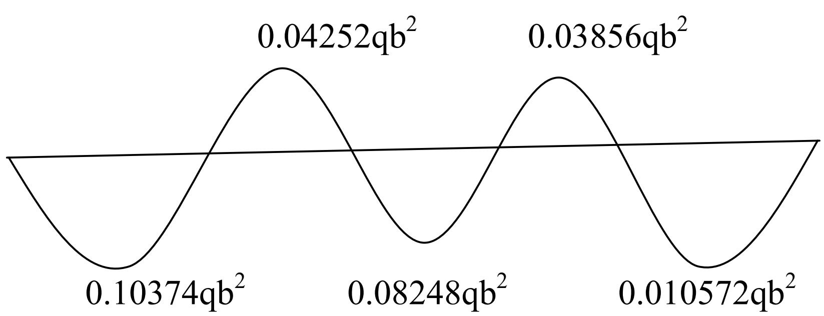


Fig.3.12: Bending Moment Diagram of Section 2-2 of two-direction continuous plate

The result of the FEM, Support Moment and span moment, for a continuous plate spanning in one direction are presented in Table 4.23 While those continuous plates in two directions are presented in Tables 4. 24- 4.27.

3.5 Matlab Programming

The programs were written using Matlab. Matlab is a strong mathematical tool used in solving complex mathematical and scientific problems. Currently, it is widely used in calculations. Matlab has tools which aid in programming through its M.files. The choice of this program language, was due to its availability and strong mathematical and scientific capability. Matlab M.file environment can be manipulated easily unlike other softwares like Fortrans and C++ to achieve the required results. It can shorten programs based on it inbuilt complex functions.

3.5.1 Single Panel Rectangular Plate Program

The programs to analyze each of the twelve individual plate cases for pure bending, buckling, and free vibration are presented in Appendix 1. These Programs are straight forward, written in simple terms and are easy to understand and apply.

The values gotten from these programs are presented in Tables 4.1 to 4.12. The values for amplitude (A), maximum deflection (W_{max}), and center moments (M_{xc} and M_{yc}), were gotten by substituting the respective values at the center or mid span of the plate for the first six plates without free edge. This point is considered the point of maximum deflection and moments of the plates. While for those with free edges, the deflection was considered at the midpoint of the free edge where deflection is considered maximum. The maximum moments, M_{xc} and $M_{yc,}$ were consider still at the center. The edge moments were considered at the clamp edges only, because that is where moment occurs. Shear forces were considered at the edges also.

Also, the program contains codes for the analysis of buckling and free vibration of rectangular plates based on the theory developed earlier in this chapter. The programs are develop in such a way that, for each plate cases, one can obtained results for pure bending parameters, critical buckling load, and fundamental frequency of the plate.

3.5.1.1 The Algorithm for Single Panel Rectangular Plates Program

The single panel rectangular plate programs have the following algorithms:

All integrals are from 0 to 1.

- Start
- Input the shape function, k = U*V
- Input Poisson ratio, v;
- Input plate dimensions, a and b; udl, q, plate thickness, h; young's modulus, E; and specific density of plate material, ρ.
- Calculate aspect ratio, s = b/a
- Calculate flexural rigidity, $D = \frac{Eh^3}{12(1-v^2)}$.
- Calculate coefficient of amplitude of deflection $u_s = \frac{\int\int k\,\partial R\,\partial Q}{\int\int [(k''^R)^2 + \frac{2}{S^2}(k''^{RQ})^2 + \frac{1}{S^4}(k''^Q)^2]\,\partial R\,\partial Q}$
- Input values of R and Q for maximum deflection
- Calculate amplitude of deflection, $A = u_s * \frac{qa^4}{D}$
- Calculate coefficient of deflection, $\alpha = u_s k$
- Calculate maximum Deflection ,$w = \alpha * \frac{qa^4}{D}$
- Input values of R_1 and Q_1 for maximum or center moment
- Calculate the coefficients of Maximum moment, $\beta = -u_s(\frac{\partial^2 k}{\partial R^2} + v\frac{\partial^2 k}{S^2\,\partial Q^2})$
- Calculate Maximum Moment, $M_{xmax} = \beta qa^2$
- Calculate the coefficients of Maximum moment, $\beta_1 = -u_s(v\frac{\partial^2 k}{\partial R^2} + \frac{\partial^2 k}{S\,\partial Q^2})$
- Calculate Maximum Moment, $M_{ymax} = \beta_1 qa^2$
- Input values of R_4 and Q_4 for maximum shear force in x-direction
- Calculate the coefficients of maximum shear force, $\delta = -u_s[\frac{\partial^3 k}{\partial R^3} + (2-v)\frac{\partial^3 k}{S^2\,\partial R\,\partial Q^2}]$

- Calculate Maximum shear force in x-direction, $V_{xmax} = \delta qa$
- Input values of R_5 and Q_5 for maximum shear force in y-direction
- Calculate the coefficients of maximum shear force, $\delta_1 = -u_s[\frac{\partial^3 k}{S^3\,\partial Q^3} + (2-v)\frac{\partial^3 k}{S\,\partial R^2\,\partial Q}]$
- Calculate Maximum shear force in y-direction, $V_{ymax} = \delta_1 qa$
- Calculate the buckling load factor or coefficient , $n_x =$

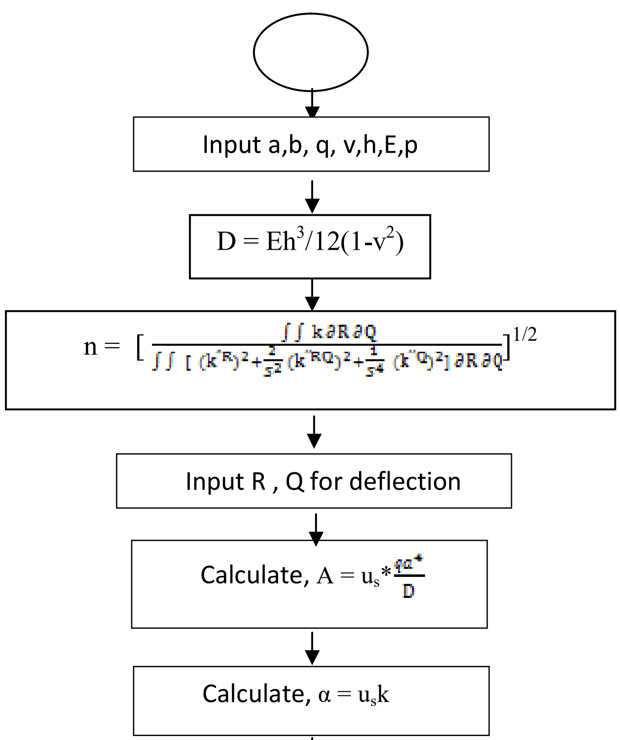
  $$\frac{\int\int [(k''^{R})^2 + \frac{2}{S^2}(k''^{RQ})^2 + \frac{1}{S^4}(k''^{Q})^2]\,\partial R\,\partial Q}{\int\int (k'^{R})^2\,\partial R\,\partial Q}$$

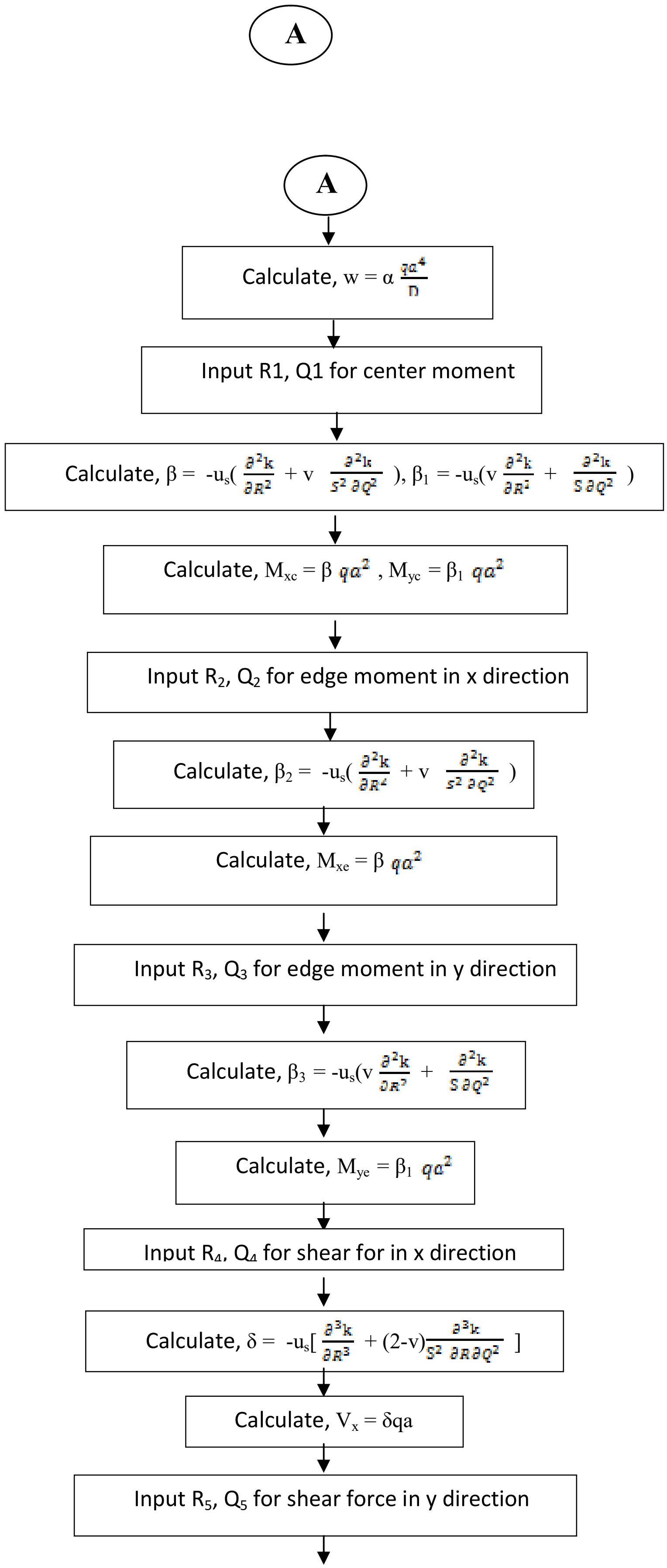
- Calculate n_x in terms of denominator b^2 , then $n_{1x} = n_x/s^2$
- Calculate $n_{2x} = n_{1x}/\pi^2$
- Calculate the Critical Buckling Load, $N_x = n_x * D/a^2$
- Calculate the coefficient of fundamental frequency, $f =$

  $$= [\frac{\int\int [(k''^{R})^2 + \frac{2}{S^2}(k''^{RQ})^2 + \frac{1}{S^4}(k''^{R})^2\,\partial R\,\partial Q}{\int\int k^2\,\partial R\,\partial Q}]^{1/2}$$

- Calculate $f_s = f/\pi^2$
- Calculate the fundamental frequency $\omega = \frac{f_s}{a^2}\sqrt{\frac{D}{\rho h}}$
- End

3.5.1.2 The Flowchart for Single Panel Rectangular Plates Program




A
A
Calculate, $w = \alpha \frac{qa^4}{D}$
Input R1, Q1 for center moment
Calculate, $\beta = -u_s(\frac{\partial^2 k}{\partial R^2} + v \frac{\partial^2 k}{S^2 \partial Q^2})$, $\beta_1 = -u_s(v \frac{\partial^2 k}{\partial R^2} + \frac{\partial^2 k}{S \partial Q^2})$
Calculate, $M_{xc} = \beta\ qa^2$, $M_{yc} = \beta_1\ qa^2$
Input R_2, Q_2 for edge moment in x direction
Calculate, $\beta_2 = -u_s(\frac{\partial^2 k}{\partial R^2} + v \frac{\partial^2 k}{S^2 \partial Q^2})$
Calculate, $M_{xe} = \beta\ qa^2$
Input R_3, Q_3 for edge moment in y direction
Calculate, $\beta_3 = -u_s(v \frac{\partial^2 k}{\partial R^2} + \frac{\partial^2 k}{S \partial Q^2}$
Calculate, $M_{ye} = \beta_1\ qa^2$
Input R_4, Q_4 for shear for in x direction
Calculate, $\delta = -u_s[\frac{\partial^3 k}{\partial R^3} + (2-v)\frac{\partial^3 k}{S^2\ \partial R \partial Q^2}]$
Calculate, $V_x = \delta qa$
Input R_5, Q_5 for shear force in y direction

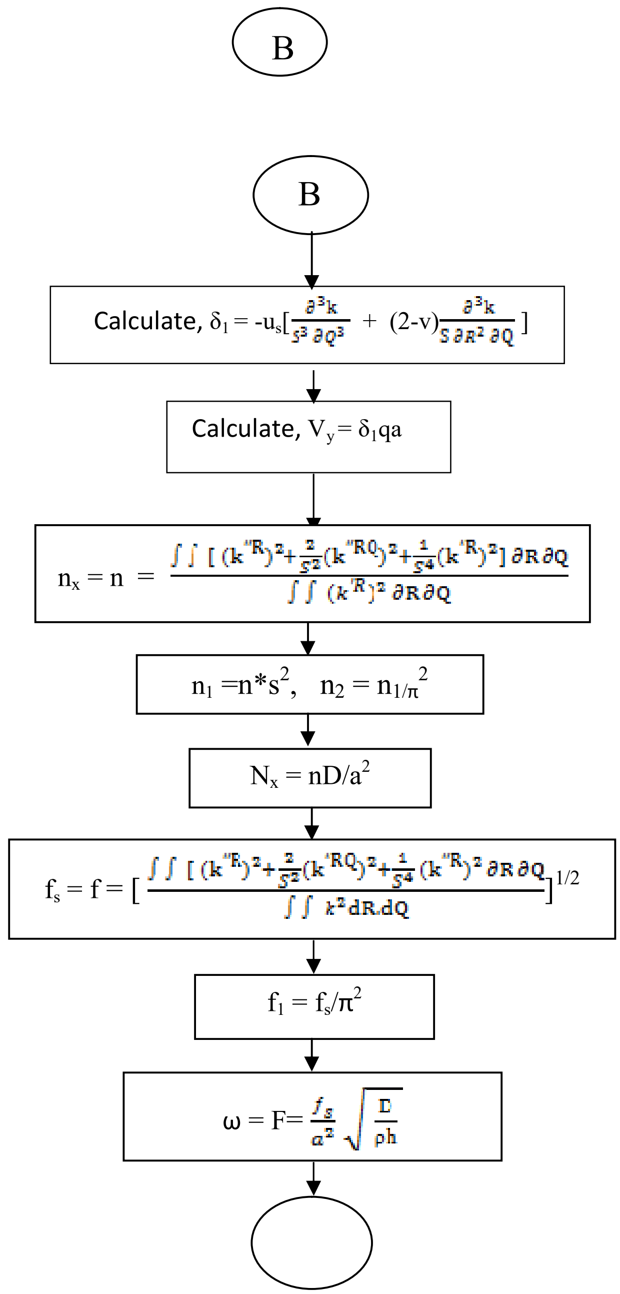


Fig.3.13: Flowchart of the Program.

3.5.2 One-Way Continuous Plate Program

The continuous plate in one-way program is presented in Appendix 2. The steps are simple to follow, clear in understanding and easy to apply. On screen messages should be followed and necessary and correct inputs should be supply by the user as required. The program follow a sequence of the analysis developed in section 3.4.1.

3.5.3 Two-Way Continuous Plate Program

Continuous plate spanning in two directions program is presented in Appendix 3. This is a lengthy program and requires diligences and consistency. It is written in clear terms and is easy to follow and apply. The program also, follow a sequence of the analysis in section 3.4.2.

In all the programs for continuous plate analysis, the user need to have a knowledge of structural mechanics especially, how to apply stiffness method in analyzing indeterminate structures, a good knowledge of rectangular plate analysis especially, using polynomial functions. Knowledge of MATLAB or programming will be an enhancer or added advantage. While working, it will be of great advantage if, the user is familiar with analysis of plate or slab manually.

CHAPTER FOUR

RESULTS AND DISCUSSION

4.1 Results

4.1.1 Results of Pure Bending Analysis

The values of the coefficients of deflection, moment and shear force from the program developed in this work for pure bending analysis for the twelve (12) plate cases under consideration, are presented in the following Tables 4.1 to 4.12:

Table 4.1: Coefficients of Amplitude, Deflection, Moments and Shear force due to uniform load over the entire area of SSSS plate for aspect ratio, s= b/a.

Aspect Ratio, S = b/a	$A = uq^4/D$	$W_{max} = \alpha qa^4/D$	Center Moment		Shear Force		Fixed Edge Moment	
			$M_{xc} = \beta qa^2$	$M_{yc} = \beta_1 qa^2$	$V_{xc} = \delta qa$	$V_{yc} = \delta_1 qa$	$M_{xe} = \beta_2 qa^2$	$M_{ye} = \beta_3 qa^2$
	u	α	β	β_1	δ	δ_1	β_2	β_3
1.0	0.04236	0.00414	0.05163	0.05163	0.07491	0.37491	0.00000	0.00000
1.2	0.05902	0.00576	0.06686	0.05502	0.43034	0.37890	0.00000	0.00000
1.5	0.08121	0.00793	0.08629	0.05668	0.48861	0.36635	0.00000	0.00000
1.6	0.08762	0.00856	0.09176	0.05673	0.50310	0.35949	0.00000	0.00000
2.0	0.10843	0.01059	0.10927	0.05591	0.54485	0.32731	0.00000	0.00000

where,

u = Coefficient of amplitude of deflection

q = Uniformly Distributed Load

α = Coefficient of deflection

β and β_1 = Cofficients of center moments in x- and y- directions respectively

β_2 and β_3 = Cofficients of edge moments in x- and y- directions respectively

δ and δ_1 = Cofficients of shear force in x- and y- directions respectively

D = Plate flexural rigidity

a = dimension along x-axis.

Table 4.2: Coefficients of Amplitude, Deflection, Moments and Shear force due to uniform load over the entire area of CCCC plate for aspect ratio, s= b/a.

Aspect Ratio S = b/a	A = uq^4/D u	W_{max} = $\alpha qa^4/D$ α	Center Moment		Shear Force		Fixed Edge Moment	
			$M_{xc} = \beta qa^2$ β	$M_{yc} = \beta_1 qa^2$ β_1	$V_{xc} = \delta qa$ δ	$V_{yc} = \delta_1 qa$ δ_1	$M_{xe} = \beta_2 qa^2$ β_2	$M_{ye} = \beta_3 qa^2$ β_3
1.0	0.34028	0.00133	0.02765	0.02765	0.25521	0.25521	-0.04252	-0.04252
1.2	0.46565	0.00182	0.03517	0.02894	0.34924	0.20211	-0.05821	-0.04042
1.5	0.60283	0.00236	0.04270	0.02804	0.45212	0.13396	-0.07535	-0.03349
1.6	0.63599	0.00248	0.04441	0.02745	0.47699	0.11645	-0.07950	-0.03105
2.0	0.72593	0.00284	0.04877	0.02495	0.54444	0.06806	-0.09074	-0.02269

Table 4.3: Coefficients of Amplitude, Deflection, Moments and Shear force due to uniform load over the entire area of CSSS plate for aspect ratio is s= b/a.

Aspect Ratio, S = b/a	A = uq^4/D u	W_{max} = $\alpha qa^4/D$ α	Center Moment		Shear Force		Fixed Edge Moment	
			$M_{xc} = \beta qa^2$ β	$M_{yc} = \beta_1 qa^2$ β_1	$V_{xmax} = \delta qa$ δ	$V_{ymax} = \delta_1 qa$ δ_1	$M_{xe} = \beta_2 qa^2$ β_2	$M_{ye} = \beta_2 qa^2$ β_3
1.0	0.07211	0.00282	0.03718	0.04191	0.29203	0.33800	0.00000	-0.06760
1.2	0.10971	0.00429	0.05185	0.04805	0.35883	0.29760	0.00000	-0.07142
1.5	0.16539	0.00646	0.07236	0.05306	0.43554	0.22971	0.00000	-0.06891
1.6	0.18241	0.00713	0.07843	0.05392	0.45532	0.20876	0.00000	-0.06680
2.0	0.23984	0.00937	0.09837	0.05509	0.51265	0.14053	0.00000	-0.05621

Table 4.4: Coefficients of Amplitude, Deflection, Moments and Shear force due to uniform load over the entire area of CSCS plate for aspect ratio, s= b/a.

Aspect Ratio, S = b/a	A = uq^4/D u	W_{max} = $\alpha qa^4/D$ α	Center Moment		Shear Force		Fixed Edge Moment	
			$M_{xc} = \beta qa^2$ β	$M_{yc} = \beta_1 qa^2$ β_1	$V_{xmax} = \delta qa$ δ	$V_{ymax} = \delta_1 qa$ δ_1	$M_{xe} = \beta_2 qa^2$ β_2	$M_{ye} = \beta_3 qa^2$ β_3
1.0	0.10180	0.00199	0.02863	0.03754	0.24941	0.38175	0.00000	-0.06363
1.2	0.16898	0.00330	0.04268	0.04618	0.32622	0.36671	0.00000	-0.07334
1.5	0.28226	0.00551	0.06469	0.05508	0.42496	0.31362	0.00000	-0.07841
1.6	0.31969	0.00624	0.07165	0.05701	0.45206	0.29268	0.00000	-0.07805

2.0	0.45335	0.00886	0.09563	0.06092	0.53269	0.21251	0.00000	-0.07084

Table 4.5: Coefficients of Amplitude, Deflection, Moments and Shear force due to uniform load over the entire area of CCSS plate for aspect ratio, s= b/a.

Aspect Ratio, S = b/a	A = uq^4/D u	W_{max} = $\alpha qa^4/D$ α	Center Moment		Shear Force		Fixed Edge Moment	
			$M_{xc} = \beta qa^2$ β	$M_{yc} = \beta_1 qa^2$ β_1	$V_{xmax} = \delta qa$ δ	$V_{ymax} = \delta_1 qa$ δ_1	$M_{xe} = \beta_2 qa^2$ β_2	$M_{ye} = \beta_3 qa^2$ β_3
1.0	0.13445	0.00210	0.03277	0.03277	0.25210	0.25210	-0.05042	-0.05042
1.2	0.18552	0.00290	0.04203	0.03459	0.34785	0.20130	-0.06957	-0.04831
1.5	0.24690	0.00386	0.05247	0.03446	0.46293	0.13716	-0.09259	-0.04115
1.6	0.26307	0.00411	0.05511	0.03407	0.49326	0.12042	-0.09865	-0.03854
2.0	0.31089	0.00486	0.06266	0.03206	0.58292	0.07287	-0.11658	-0.02915

Table 4.6: Coefficients of Amplitude, Deflection, Moments and Shear force due to uniformly load over the entire area of CCCS plate for aspect ratio, s= b/a.

Aspect Ratio S = b/a	A = uq^4/D u	W_{max} = $\alpha qa^4/D$ α	Center Moment		Shear Force		Fixed Edge Moment	
			$M_{xc} = \beta qa^2$ β	$M_{yc} = \beta_1 qa^2$ β_1	$V_{xmax} = \delta qa$ δ	$V_{ymax} = \delta_1 qa$ δ_1	$M_{xe} = \beta_2 qa^2$ β_2	$M_{ye} = \beta_3 qa^2$ β_3
1.0	0.20569	0.00161	0.02700	0.03150	0.19283	0.30854	-0.03857	-0.05142
1.2	0.31120	0.00243	0.03728	0.03577	0.29175	0.27013	-0.05835	-0.05403
1.5	0.45456	0.00355	0.05019	0.03804	0.42615	0.20203	-0.08523	-0.05051
1.6	0.49475	0.00387	0.05363	0.03807	0.46383	0.18118	-0.09277	-0.04832
2.0	0.61721	0.00482	0.06365	0.03665	0.57864	0.11573	-0.1157	-0.03858

Table 4.7: Coefficients of Amplitude, Deflection, Moments and Shear forcedue to uniformly load over the entire area of SSFS plate for aspect ratio, s= b/a.

Aspect Ratio , S = b/a	A = uq^4/D u	W_{max} = $\alpha qa^4/D$ α	Center Moment		Shear Force		Fixed Edge Moment	
			$M_{xc} = \beta qa^2$ β	$M_{yc} = \beta_1 qa^2$ β_1	$V_{xmax} = \delta qa$ δ	$V_{ymax} = \delta_1 qa$ δ_1	$M_{xe} = \beta_2 qa^2$ β_2	$M_{ye} = \beta_3 qa^2$ β_3
1.0	0.02659	0.01105	0.08007	0.04293	0.40839	0.48219	0.00000	0.00000
1.2	0.03008	0.01250	0.08843	0.04138	0.42293	0.40667	0.00000	0.00000
1.5	0.03353	0.01393	0.09659	0.03957	0.43570	0.32767	0.00000	0.00000
1.6	0.03435	0.01428	0.09853	0.03910	0.43857	0.30753	0.00000	0.00000
2.0	0.03667	0.01524	0.10397	0.03771	0.44626	0.24652	0.00000	0.00000

Table 4.8: Coefficients of Amplitude, Deflection, Moments and Shear force due to uniformly load over the entire area of SCFS plate for aspect ratio, s= b/a.

Aspect Ratio, S = b/a	A = uq^4/D u	W_{max} = $\alpha qa^4/D$ α	Center Moment		Shear Force		Fixed Edge Moment	
			M_{xc} = βqa^2 β	M_{yc} = $\beta_1 qa^2$ β_1	V_{xmax} =δqa δ	V_{ymax} =$\delta_1$qa δ_1	M_{xe} = $\beta_2 qa^2$ β_2	M_{ye} = $\beta_3 qa^2$ β_3
1.0	0.03299	0.00547	0.04877	0.02989	0.45688	0.27777	-0.09138	0.00000
1.2	0.03525	0.00586	0.05123	0.02233	0.48935	0.22550	-0.09787	0.00000
1.5	0.03733	0.00621	0.05339	0.02074	0.51834	0.17553	-0.10367	0.00000
1.6	0.03780	0.00628	0.05387	0.02036	0.52489	0.16346	-0.10498	0.00000
2.0	0.03908	0.00650	0.05517	0.01933	0.54253	0.12829	-0.10851	0.00000

Table 4.9: Coefficients of Amplitude, Deflection, Moments and Shear force due to uniformly load over the entire area of CSFS plate for aspect ratio, s= b/a.

Aspect Ratio, S = b/a	A = uq^4/D u	W_{max} = $\alpha qa^4/D$ α	Center Moment		Shear Force		Fixed Edge Moment	
			Mx_c = βqa^2 β	My_c = $\beta_1 qa^2$ β_1	Vx_{max} =δqa δ	Vy_{max} =$\delta_1$qa δ_1	Mxe = $\beta_2 qa^2$ β_2	Mye = $\beta_3 qa^2$ β_3
1.0	0.07073	0.00884	0.06167	0.04062	0.34975	1.69961	0.00000	-0.01238
1.2	0.08749	0.01094	0.07352	0.04106	0.36370	1.5347	0.00000	-0.01063
1.5	0.10408	0.01301	0.08478	0.03990	0.37769	1.2915	0.00000	-0.08095
1.6	0.10796	0.01350	0.0873	0.03939	0.38127	1.2205	0.00000	-0.0738
2.0	0.11853	0.01482	0.09417	0.03752	0.39218	0.09907	0.00000	-0.05186

Table 4.10: Coefficients of Amplitude, Deflection, Moments and Shear force due to uniformly load over the entire area of CCFS plate for aspect ratio, s= b/a.

Aspect Ratio S = b/a	A = uq^4/D u	W_{max} = $\alpha qa^4/D$ α	Center Moment		Shear Force		Fixed Edge Moment	
			M_{xc} = βqa^2 β	M_{yc} = $\beta_1 qa^2$ β_1	Vx_{max} =δqa δ	V_{ymax} =$\delta_1$qa δ_1	M_{xe} = $\beta_2 qa^2$ β_2	M_{ye} = $\beta_2 qa^2$ β_3
1.0	0.09696	0.00485	0.04127	0.02451	0.37269	1.07044	-0.07454	-0.06787
1.2	0.10953	0.00548	0.04524	0.02309	0.42100	0.89889	-0.0842	-0.05324
1.5	0.12015	0.00601	0.04834	0.02120	0.46183	0.71076	-0.09237	-0.03738
1.6	0.12243	0.00612	0.04903	0.02069	0.47058	0.66290	-0.09412	-0.03348
2.0	0.12828	0.00641	0.05063	0.0192	0.49308	0.52051	-0.09862	-0.02245

Table 4.11: Coefficients of Amplitude, Deflection, Moments and Shear force due to uniformly load over the entire area of SCFC plate for aspect ratio, s= b/a.

Aspect Ratio, S = b/a	$A = uq^4/D$	$W_{max} = \alpha qa^4/D$	Center Moment		Shear Force		Fixed Edge Moment	
	u	α	$M_{xc} = \beta qa^2$ β	$M_{yc} = \beta_1 qa^2$ β_1	$V_{xmax} = \delta qa$ δ	$V_{ymax} = \delta_1 qa$ δ_1	$M_{xe} = \beta_2 qa^2$ β_2	$M_{ye} = \beta_3 qa^2$ β_3
1.0	0.03672	0.00305	0.03571	0.01593	0.40781	0.19132	-0.06797	0.00000
1.2	0.03818	0.00317	0.036524	0.01472	0.42340	0.15339	-0.07057	0.00000
1.5	0.03930	0.00327	0.03720	0.01364	0.43654	0.11834	-0.07276	0.00000
1.6	0.03956	0.00329	0.03734	0.01349	0.43941	0.11001	-0.07324	0.00000
2.0	0.04023	0.00335	0.03772	0.01275	0.44700	0.08599	-0.07450	0.00000

Table 4.12: Coefficients of Amplitude, Deflection, Moments and Shear force due to uniformly load over the entire area of CCFC plate for aspect ratio, s= b/a.

Aspect Ratio , S = b/a	$A = uq^4/D$	$W_{max} = \alpha qa^4/D$	Center Moment		Shear Force		Fixed Edge Moment	
	u	α	$M_{xc} = \beta qa^2$ β	$M_{yc} = \beta_1 qa^2$ β_1	$V_{xmax} = \delta qa$ δ	$V_{ymax} = \delta_1 qa$ δ_1	$M_{xe} = \beta_2 qa^2$ β_2	$M_{ye} = \beta_2 qa^2$ β_3
1.0	0.1143	0.00286	0.03165	0.01664	0.35146	0.76693	-0.05858	-0.04000
1.2	0.12235	0.00306	0.03310	0.01525	0.37623	0.62339	-0.06270	-0.02974
1.5	0.12856	0.00321	0.03412	0.01381	0.39533	0.48225	-0.06589	-0.02000
1.6	0.12983	0.00325	0.03432	0.01347	0.39923	0.44806	-0.06654	-0.01775
2.0	0.133	0.00333	0.03477	0.01251	0.40898	0.34896	-0.06816	-0.01164

4.1.2 Results of Buckling Analysis

The values of the coefficients of critical buckling load from the developed program for the twelve (12) plate cases under consideration, are presented on the following Tables 4.13 -4.18:

Table 4.13: Critical Buckling Load Coefficients for SSSS and CCCC plates

Aspect Ratio, S = b/a	SSSS plate			CCCC plate		
	$N_x = n_x \frac{D}{a^2}$; n_x	$N_x = n_{1x} \frac{D}{b^2}$; n_{1x}	$N_x = n_{2x} \frac{D}{b^2}$; n_{2x}	$N_x = n_x \frac{D}{a^2}$; n_x	$N_x = n_{1x} \frac{D}{b^2}$; n_{1x}	$N_x = n_{2x} \frac{D}{b^2}$; n_{2x}

1.0	39.508	39.508	4.003	108.000	108.000	10.943
1.2	28.355	40.831	4.137	78.921	113.647	11.515
1.5	20.608	46.367	4.698	60.963	137.167	13.898
1.6	19.102	48.901	4.955	57.784	147.926	14.988
2.0	15.435	61.742	6.256	50.625	2.2.500	20.518

where,

n_x is the coefficient of critical buckling load of the plate for aspect ratio, s = b/a

n_{1x} is the coefficient of critical buckling load of the plate for aspect ratio, p = a/b

$n_{2x} = n_{1x}/\pi^2$

D = Plate flexural rigidity

a and b = dimensions in x- and y- directions respectively

Table 4.14: Critical Buckling Load Coefficients for CSSS and CSCS plates

Aspect Ratio; S = b/a	CSSS Plate			CSCS Plate		
	$N_x = n_x\frac{D}{a^2}$; n_x	$N_x = n_{1x}\frac{D}{b^2}$; n_{1x}	$N_x = n_{2x}\frac{D}{b^2}$; n_{2x}	$N_x = n_x\frac{D}{a^2}$; n_x	$N_x = n_{1x}\frac{D}{b^2}$; n_{1x}	$N_x = n_{2x}\frac{D}{b^2}$; n_{2x}
1.0	56.805	56.805	5.756	84.941	84.941	8.606
1.2	37.336	53.763	5.447	51.172	73.688	7.466
1.5	24.765	55.721	5.646	30.635	68.928	6.984
1.6	22.454	57.483	5.824	27.048	69.244	7.016
2.0	17.078	68.313	6.922	19.074	76.294	7.730

n_x, n_{1x}, n_{2x}, D, a and b are as defined in Table 4.13

Table 4.15: Critical Buckling Load Coefficients for CCSS and CCCS plates

Aspect Ratio; S = b/a	CCSS Plate			CCCS Plate		
	$N_x = n_x\frac{D}{a^2}$; n_x	$N_x = n_{1x}\frac{D}{b^2}$; n_{1x}	$N_x = n_{2x}\frac{D}{b^2}$; n_{2x}	$N_x = n_x\frac{D}{a^2}$; n_x	$N_x = n_{1x}\frac{D}{b^2}$ n_{1x}	$N_x = n_{2x}\frac{D}{b^2}$ n_{2x}
1.0	64.737	64.737	6.559	89.333	89.333	9.051
1.2	46.917	67.560	6.845	59.047	85.027	8.616
1.5	35.253	79.320	8.037	40.424	90.954	9.216
1.6	33.086	84.700	8.582	37.140	95.078	9.633
2.0	27.997	111.987	11.347	29.771	119.083	12.066

n_x, n_{1x}, n_{2x}, D, a and b are as defined in Table 4.13

Table 4.16: Critical Buckling Load Coefficients for SSFS and SCFS plates

Aspect Ratio; S = b/a	SSFS			SCFS		
	$N_x = n_x \frac{D}{a^2}$; n_x	$N_x = n_{1x} \frac{D}{b^2}$; n_{1x}	$N_x = n_{2x} \frac{D}{b^2}$; n_{2x}	$N_x = n_x \frac{D}{a^2}$; n_x	$N_x = n_{1x} \frac{D}{b^2}$; n_{1x}	$N_x = n_{2x} \frac{D}{b^2}$; n_{2x}
1.0	15.415	15.415	1.562	26.472	26.472	2.682
1.2	13.627	19.622	1.988	24.715	35.589	3.606
1.5	12.227	27.512	2.788	23.333	52.499	5.319
1.6	11.934	30.550	3.095	23.042	58.988	5.977
2.0	11.179	44.716	4.531	22.293	89.171	9.035

n_x, n_{1x}, n_{2x}, D, a and b are as defined in Table 4.13

Table 4.17: Critical Buckling Load Coefficients for CSFS and CCFS plates

Aspect Ratio; S= b/a	CSFS Plate			CCFS Plate		
	$N_x = n_x \frac{D}{a^2}$; n_x	$N_x = n_{1x} \frac{D}{b^2}$; n_{1x}	$N_x = n_{2x} \frac{D}{b^2}$; n_{2x}	$N_x = n_x \frac{D}{a^2}$ n_x	$N_x = n_{1x} \frac{D}{b^2}$ n_{1x}	$N_x = n_{2x} \frac{D}{b^2}$ n_{2x}
1.0	19.283	19.283	1.957	29.891	29.891	3.029
1.2	15.589	22.448	2.275	26.461	38.104	3.861
1.5	13.105	29.485	2.987	24.122	54.274	5.499
1.6	12.633	32.341	3.277	23.673	60.603	6.140
2.0	11.507	46.023	4.664	22.593	90.370	9.156

n_x, n_{1x}, n_{2x}, D, a and b are as defined in Table 4.13

Table 4.18: Critical Buckling Load Coefficients for SCFC and CCFC Plate

Aspect Ratio; S= b/a	SCFC Plate			CCFC Plate		
	$N_x = n_x \frac{D}{a^2}$; n_x	$N_x = n_{1x} \frac{D}{b^2}$; n_{1x}	$N_x = n_{2x} \frac{D}{b^2}$; n_{2x}	$N_x = n_x \frac{D}{a^2}$; n_x	$N_x = n_{1x} \frac{D}{b^2}$; n_{1x}	$N_x = n_{2x} \frac{D}{b^2}$; n_{2x}
1.0	47.451	47.451	4.808	50.714	50.714	5.139
1.2	45.705	65.815	6.668	47.376	68.221	6.912
1.5	44.329	99.740	10.106	45.087	101.445	10.279
1.6	44.039	112.739	11.423	44.646	114.294	11.580
2.0	43.291	173.166	17.545	43.581	174.326	17.663

n_x, n_{1x}, n_{2x}, D, a and b are as defined in Table 4.13

4.1.3 Results of Free Vibration Analysis

The values of the coefficients of the fundamental natural frequency, ω, from the Program written for the twelve (12) plate cases under consideration, are presented in Tables 4.19-4.22:

Table 4.19: Coefficients of Fundamental Natural Frequency, ω, for SSSS, CCCC and CSSS Plates.

Aspect Ratio ; S = b/a	SSSS Plate		CCCC Plate		CSSS Plate	
	$\omega = \frac{f_s}{a^2}\sqrt{\frac{D}{\rho h}}$; f_S	$\omega = \frac{f_{1s}}{a^2}\sqrt{\frac{D}{\rho h}}$; $f_{1S} = f_s/\pi^2$	$\omega = \frac{f_s}{a^2}\sqrt{\frac{D}{\rho h}}$; f_S	$\omega = \frac{f_{1s}}{a^2}\sqrt{\frac{D}{\rho h}}$; $f_{1S} = f_s/\pi^2$	$\omega = \frac{f_s}{a^2}\sqrt{\frac{D}{\rho h}}$; f_S	$\omega = \frac{f_{1s}}{a^2}\sqrt{\frac{D}{\rho h}}$; $f_{1S} = f_s/\pi^2$
1.0	19.749	2.001	36.000	3.648	23.680	2.399
1.2	16.729	1.695	30.774	3.118	19.197	1.945
1.5	14.262	1.445	27.047	2.740	15.635	1.584
1.6	13.732	1.391	26.333	2.668	14.888	1.508
2.0	12.344	1.251	24.648	2.497	12.984	1.316

where f_s is the coefficient of the fundamental natural frequency

$f_{1S} = f_s/\pi^2$

D = Plate Flexural rigidity

a = dimension along x-axis

h = plate thickness

ρ = specific density of the plate

Table 4.20: Coefficients of Fundamental Natural Frequency, ω, CSCS, CCSS and CCCS Plate.

Aspect Ratio; S= b/a S	CSCS Plate		CCSS Plate		CCCS Plate	
	$\omega = \frac{f_s}{a^2}\sqrt{\frac{D}{\rho h}}$; f_S	$\omega = \frac{f_{1s}}{a^2}\sqrt{\frac{D}{\rho h}}$; $f_{1S} = f_s/\pi^2$	$\omega = \frac{f_s}{a^2}\sqrt{\frac{D}{\rho h}}$; f_S	$\omega = \frac{f_{1s}}{a^2}\sqrt{\frac{D}{\rho h}}$; $f_{1S} = f_s/\pi^2$	$\omega = \frac{f_s}{a^2}\sqrt{\frac{D}{\rho h}}$; f_S	$\omega = \frac{f_{1s}}{a^2}\sqrt{\frac{D}{\rho h}}$; $f_{1S} = f_s/\pi^2$
1.0	28.956	2.934	27.129	2.749	31.868	3.229
1.2	22.475	2.277	23.095	2.340	25.909	2.625
1.5	17.389	1.762	20.019	2.028	21.437	2.172
1.6	16.340	1.656	19.394	1.965	20.548	2.082

2.0	13.721	1.390	17.840	1.808	18.397	1.864

where f_s, f_{1S}, D, a, h, and ρ are as defined in Table 19

Table 4.21: Coefficients of Fundamental Natural Frequency, ω, SSFS, SCFS and CSFS Plate.

Aspect Ratio; S= b/a	SSFS Plate		SCFS Plate		CSFS Plate	
	$\omega = \frac{f_s}{a^2}\sqrt{\frac{D}{\rho h}}$;	$\omega = \frac{f_{1s}}{a^2}\sqrt{\frac{D}{\rho h}}$;	$\omega = \frac{f_s}{a^2}\sqrt{\frac{D}{\rho h}}$;	$\omega = \frac{f_{1s}}{a^2}\sqrt{\frac{D}{\rho h}}$;	$\omega = \frac{f_s}{a^2}\sqrt{\frac{D}{\rho h}}$;	$\omega = \frac{f_{1s}}{a^2}\sqrt{\frac{D}{\rho h}}$;
S	f_S	$f_{1S} = f_s/\pi^2$	f_S	$f_{1S} = f_s/\pi^2$	f_S	$f_{1S} = f_s/\pi^2$
1.0	12.335	1.250	17.348	1.758	13.797	1.398
1.2	11.598	1.175	16.762	1.698	12.405	1.257
1.5	10.986	1.113	16.287	1.650	11.374	1.152
1.6	10.853	1.100	16.185	1.640	11.167	1.131
2.0	10.505	1.064	15.920	1.613	10.658	1.080

where f_s, f_{1S}, D, a, h, and ρ are as defined in Table 19

Table 4.22: Coefficients of Fundamental Natural Frequency, ω, for CCFS, SCFC and CCFC Plate

Aspect Ratio; S= b/a	CCFS Plate		SCFC Plate		CCFC Plate	
	$\omega = \frac{f_s}{a^2}\sqrt{\frac{D}{\rho h}}$;	$\omega = \frac{f_{1s}}{a^2}\sqrt{\frac{D}{\rho h}}$;	$\omega = \frac{f_s}{a^2}\sqrt{\frac{D}{\rho h}}$;	$\omega = \frac{f_{1s}}{a^2}\sqrt{\frac{D}{\rho h}}$;	$\omega = \frac{f_s}{a^2}\sqrt{\frac{D}{\rho h}}$;	$\omega = \frac{f_{1s}}{a^2}\sqrt{\frac{D}{\rho h}}$;
	f_S	$f_{1S} = f_s/\pi^2$	f_S	$f_{1S} = f_s/\pi^2$	f_S	$f_{1S} = f_s/\pi^2$
1.0	18.434	1.868	23.863	2.418	24.669	2.500
1.2	17.344	1.757	23.419	2.373	23.843	2.416
1.5	16.560	1.678	23.064	2.337	23.260	2.357
1.6	16.405	1.662	22.988	2.329	23.146	2.345
2.0	16.026	1.624	22.793	2.309	22.869	2.317

where f_s, f_{1S}, D, a, h, and ρ are as defined in Table 19

4.1.4 Results of Analysis of Continuous Plate Spanning in one way

The results of fixed edge moment (FEM) and support moment (SPTM) obtained from manual analysis and computer program developed of a one-way four span continuous plate for aspect ratio (S =b/a = 1), are presented in the Tables 4.23 for aspect ratio.

Table 4.23: Results of Fixed edge moment (FEM) and Support Moment (SPTM) obtained from manual analysis and from computer program for one-way four span continuous plate. s= 1.

FEM = $\beta_1 qa^2$; β_1	FEM = $\beta_2 qa^2$; β_2	SPTM= $\beta_3 qa^2$; β_3	SPTM = $\beta_4 qa^2$; β_4
0.0000	0.0000	0.00000	0.00000
-0.06760	-0.06760	-0.06590	-0.06590
-0.06363	-0.06363	-0.06136	-0.06135
-0.06363	-0.06363	-0.06250	-0.06249
-0.06363	-0.06363	-0.06476	-0.06476
-0.06363	-0.06363	-0.06590	-0.06590
-0.06760	-0.06760	-0.06930	-0.06930
0.0000	-0.00000	0.00000	0.00000

Where,

β_1 and β_3 are the cofficients of fixed end moment and support moment respectively obtained from manual approach. And β_2 and β_4 are the cofficients of fixed end moment and support moment respectively obtained from the developed program.

4.1.5 Results of Analysis of Continuous Plate Spanning in Two-Way

The results of fixed edge moment (FEM) and support moment (SPTM) obtained from manual analysis and computer program developed of a two-way four by three span continuous plate for aspect ratio (S =b/a = 1), are presented in the Tables 4.24-4.27.

Table 4.24: Results of Fixed End Moment (FEM) and Support Moment (SPTM) obtained from manual analysis and computer program for two-way four span continuous plate at Section S-S. S = 1

FEM = $\beta_1 qa^2$; β_1	FEM = $\beta_2 qa^2$; β_2	SPTM = $\beta_3 qa^2$; β_3	SPTM = $\beta_4 qa^2$; β_4
0.0000	0.0000	0.00000	0.0000
-0.05042	-0.05042	-0.05085	-0.05085
-0.05142	-0.05142	-0.05199	-0.05199
-0.05142	-0.05142	-0.05171	-0.05171
-0.05142	-0.05142	-0.05113	-0.05114
-0.05142	-0.05142	-0.05085	-0.05085
-0.05042	-0.05042	-0.04999	0.04999
0.0000	0.0000	0.00000	0.0000

Where,

β_1 and β_3 are the cofficients of fixed end moment and support moment respectively obtained from manual approach. And β_2 and β_4 are the cofficients of fixed end moment and support moment respectively obtained from the developed program.

Table 4.25: Results of Fixed End Moment (FEM) and Support Moment (SPTM) obtained from manual analysis and computer program for two-direction four span continuous plate at Section T-T. S = 1

FEM = $\beta_1 qa^2$; β_1	FEM = $\beta_2 qa^2$; β_2	SPTM = $\beta_3 qa^2$; β_3	SPTM = $\beta_4 qa^2$; β_4
0.0000	0.0000	0.00000	0.0000
-0.03856	-0.03857	-0.04026	-0.04027
-0.04252	-0.04253	-0.04478	-0.04480
-0.04252	-0.04253	-0.04365	-0.04367
-0.04252	-0.04253	-0.04139	-0.04140
-0.04252	-0.04253	-0.04024	-0.04027
-0.03856	-0.0387	-0.03686	-0.03686

0.0000	0.0000	0.00000	0.0000

Table 4.26: Results of Fixed End Moment (FEM) and Support Moment (SPTM) obtained from manual analysis and computer program for two-direction three span continuous plate at Section 1-1. S = 1

FEM = $\beta_1 qa^2$; β_1	FEM= $\beta_2 qa^2$; β_2	SPTM = $\beta_3 qa^2$; β_3	SPTM = $\beta_4 qa^2$; β_4
0.0000	0.00000	0.00000	0.00000
-0.05042	-0.05042	-0.05102	-0.05102
-0.05142	-0.05142	-0.05182	-0.05182
-0.05142	-0.05142	-0.05102	-0.05102
-0.05042	-0.05042	-0.04982	0.04982
0.00000	0.00000	0.00000	0.00000

Table 4.27: Results of Fixed End Moment (FEM) and Support Moment (SPTM) obtained from manual analysis and computer program for two-direction three span continuous plate at Section 2-2. S = 1

FEM = $\beta_1 qa^2$; β_1	FEM= $\beta_2 qa^2$; β_2	Percentage Difference $100(\beta_2 - \beta_1)/\beta_1$	SPTM = $\beta_3 qa^2$ β_3	SPTM = $\beta_4 qa^2$ β_4	Percentage Difference $100(\beta_4 - \beta_3)/\beta_3$
0.0000	0.00000	0.00	0.00000	0.00000	0.00
-0.03856	-0.03857	0.026	-0.04094	-0.04095	0.024
-0.04252	-0.04253	0.024	-0.04410	-0.04412	0.045
-0.04252	-0.04253	0.024	-0.04094	-0.04095	0.024
-0.03856	-0.03857	0.026	-0.03618	-0.03618	0.00
0.00000	0.00000	0.00	0.00000	0.00000	0.00

4.2 Discussion of Results

4.2.1 Discussion of the Results of Pure Bending Analysis

In other to validate the solutions of this present study (i.e from the program), comparison was made between the results of this work for aspect ratios 1.0, 1.2, 1.5, 1.6 and 2.0, with the solutions obtained from existing research works that used classical and approximate methods. Preference was given to the works of

Ibearugbulem et al, who used the same polynomial functions and whose results are already established as adequate.

Table 4.28: Comparison of coefficients of amplitude of deflection 'u' from the developed SSSS plate program and those of Ibearugbulem et al. (2013)

Aspect Ratio ; S = b/a	Present study $A = uq^4/D$; u	Ibearugbulem et al.(2013) $A = uq^4/D$; u_1	% difference $100(u- u_1)/ u_1$
1.0	0.04236	0.04236	0.00
1.2	0.05902	0.05902	0.00
1.5	0.08121	0.08121	0.00
1.6	0.08762	0.08762	0.00
2.0	0.10843	0.10843	0.00
Aver. %diff.			0.00

Where A is amplitude of deflection

The amplitude coefficient, 'u' at the center of the plate, obtained from this developed program in column 2, of Table 4.28, were compared with those obtained by Ibearugbulem et al. (2013), who used direct integration and work principles in their work. The percentage difference obtained from $100(z_a-z_b)/z_b$ (i.e the general expression of percentage difference, where z, is an arbitral viarable representing coefficient of any viable in consideration, and subscript, a, b represent any number, 0,1, 2, … etc, for example, here, z = u, a = 0 and b = 1), for aspect ratios 1.0, 1.2, 1.5, 1.6, and 2.0 are 0.000%, 0.000%, 0.000%, 0.000% and 0.000% respectively. These indicate that the results obtained from the present study, are accurate and adequate.

Table 4.29: Comparison of coefficients of deflection 'α' obtained from the developed SSSS plate program with those of Ibearugbulem et al. (2013), Timoshenko and Woinowsky-Krieger (1959) and Ventsel and Krauthammer (2001)

Aspect Ratio; S = b/a	Present study $W_{max} = \alpha qa^4/D$; α	Ibearugbulem et al.(2013) $W_{max} = \alpha qa^4/D$; α_1	% difference $100(\alpha- \alpha_1)/\alpha_1$	Timoshenko et al.(1959) $W_{max} = \alpha qa^4/D$; α_2	% difference $100(\alpha- \alpha_2)/\alpha_2$	Ventsel et al.(2001) $W_{max} = \alpha qa^4/D$; α_3	% difference $100(\alpha- \alpha_3)/\alpha_3$
1.0	0.00414	0.00414	0.00	0.06004	1.970	0.00416	-2.479
1.2	0.00576	0.00576	0.00	0.00564	2.128	0.00580	-2.775
1.5	0.00793	0.00793	0.00	0.00772	2.720	0.00798	-3.324

1.6	0.00856	0.00856	0.00	0.00830	3.133	0.00861	-3.687
2.0	0.01059	0.01059	0.00	0.01013	4.541	0.01065	-0.563
Aver. %diff.			0.00		2.898		-2.5656

Also, comparing the values of the deflection coefficient, α, at the center of the plate, obtained from the present study (i.e developed program) with those obtained by Ibearugbulem et al. (2013) in Table 4.29, the percentage difference for aspect ratios 1.0, 1.2, 1.5, 1.6, and 2.0 are 0.000%, 0.000%, 0.000%, 0.000% and 0.000% respectively. This indicates that there is no difference between the two results. Comparing the same results with those obtained by Timoshenko and Woinowsky-Krieger (1959), the percentage difference for the same aspect ratios, given above are 1.970%, 2.128%, 2.720%, 3.133%, and 4.541% respectively; these are all less than 5%. Thus, it agrees with the fact that the values obtained from this program are close to the exact values and are upper bound to those values obtained from existing research. To further validate these values, comparison was made with those values obtained by Ventsel and Krauthammer (2001). Here, the percentage differences for the same aspect ratios are -2.479%, -2.775%, -3.324%, -3.687%, and -0.56338% respectively, which indicate that the values from the present work (or PROGRAM) are lower bound to those of Ventsel and Krauthammer (2001). However, both of them are close. The results of the present study, lies between two of the results from existing research works, which further confirm the accuracy of the results obtained, and hencethe adequacy of the program. From the Table 4.29, it is seen that deflection increases with increase in aspect ratio.

Table 4.30: Comparison of coefficients of bending moment, 'β' obtained from the developed SSSS plate program with those of Ibearugbulem et al. (2013)

Aspect Ratio S = b/a;	Present study $M_{xc} = \beta qa^2$; β	Ibearugbulem et al.(2013) $M_{xc} = \beta qa^2$; β_1	% difference $100(\beta - \beta_1)/\beta_1$	Present study $M_{yc} = \beta_1 qa^2$; β_2	Ibearugbulem et al.(2013) $M_{yc} = \beta_1 qa^2$; β_3	% difference $100(\beta_2 - \beta_3)/\beta_3$
1.0	0.05163	0.05163	0.00	0.05163	0.05163	0.00
1.2	0.06686	0.06686	0.00	0.05502	0.05502	0.00
1.5	0.08629	0.08629	0.00	0.05668	0.05668	0.00
1.6	0.09176	0.09177	-0.01	0.05673	0.05673	0.00
2.0	0.10927	0.10927	0.00	0.05591	0.05591	0.00
Aver. %diff.			-0.002			0.00

The percentage difference between thenewly determined values of the moment M_{xc} at the center of the plate for the aspect ratios indicated and those values obtained by Ibearugbulem et al. (2013) (see Table 4.30), indicate that, the maximum percentage difference is -0.01%. This is insignificant, and hence it indicates that, these new values obtained in this study, are very close to those values they compared with. In a similar way, moment in y-direction (M_{yc}) given in Table 4.30, indicate insignificant difference. The moments at the center of the plate in x-direction increases with increase in aspect ratio, while the center moment in the y- direction, increased steadily with increase in aspect ratio, but decreases slightly with aspect ratio of 2.

Table 4.31 Comparison of coefficients of shear force 'δ' obtained from the developed SSSS plate program with those of Ibearugbulem et al. (2014)

Aspect Ratio; S = b/a	Present study $V_{xmax} = \delta qa$; δ	Ibearugbulem et al(2014) $V_{xmax} = \delta qa$; δ_1	% difference $100(\delta - \delta_1)/\delta_1$	Present study $V_{ymax} = \delta_1 qa$; δ_2	Ibearugbulem et al(2014) $V_{ymax} = \delta_1 qa$; δ_3	% difference $100(\delta_2 - \delta_3)/\delta_3$
1.0	0.37491	0.37491	0.00	0.37491	0.37491	0.00
1.2	0.43034	0.43033	0.002	0.37890	0.37890	0.00
1.5	0.48861	0.48860	0.002	0.36635	0.36634	0.003
1.6	0.50310	0.50309	0.002	0.35949	0.35948	0.003
2.0	0.54485	0.54482	0.006	0.32731	0.3273	0.003
Aver. %diff.			0.002			0.002

Comparing the values of the shear forces, V_x in x-direction obtained in this work with those obtained by Ibearugbulem et al. (2014) for the same five aspect ratios gave percentage differences between them as 0.000%, 0.002%, 0.002%, 0.002% and 0.006% respectively which indicate that the values are close to those compared with. And for shear force, V_y in y-direction, 0.000%, 0.000%, 0.003%, 0.003% and 0.003% respectively were obtained as percentage differences, which also indicate the closeness of the values. The shear force in x- direction increased with increase in aspect ratio, while those in y- direction decreased with increase in aspect ratio.

The moments at the edges of a simply supported plate, are zero given that a simple support, does not develop or have moment.

Hence, the analysis, indicates that the results of present study (i.e program) are reliable and adequate, and thus, provides a quicker approach for the analysis of rectangular SSSS plate loaded uniformly over the entire surface.

In table 4.32, thecomputer results of the CCCC plate analyzed in bending were compared with results of pure bending analysis of the CCCC plate obtained by some scholars based on exact and approximate methods.

Table 4.32: Comparison of coefficients of deflection 'α' obtained from the developed CCCC plate program with those of Ibearugbulem et al. (2014).

Aspect Ratio; S = b/a	Present study $W_{max} = \alpha qa^4/D$; α	Ibearugbulem et al.(2014) $W_{max} = \alpha qa^4/D$; α_1	% difference $100(\alpha - \alpha_1)/\alpha_1$	Timoshenko et al.(1959) $W_{max} = \alpha qa^4/D$; α_2	% difference $100(\alpha - \alpha_2)/\alpha_2$
1.0	0.00133	0.00133	0.00	0.00126	5.556
1.2	0.00182	0.00182	0.00	0.00172	5.814
1.5	0.00236	0.00236	0.00	0.00220	7.273
1.6	0.00248	0.00249	-0.40	0.00230	7.826
2.0	0.00284	0.00284	0.00	0.00254	11.811
Aver. %diff.			-0.080		7.656

Comparison of the deflection coefficients, α, at the center of the plate, obtained from the present study with those obtained by Ibearugbulem et al. (2014), yielded percentage differences of 0.000%, 0.000%, 0.000%, -0.400% and 0.000% respectively for aspect ratios 1.0, 1.2, 1.5, 1.6, and 2.0. The average pencentage dofference, was -0.080%. These values indicate that they agree closely with each other. Comparing the same values with those obtained by Timoshenko and Woinowsky-Krieger (1959), gave percentage differences of 5.556%, 5.814%, 7.273%, 7.826%, and 11.811% respectively for the same aspect ratio. All the values are above 5%, but are within acceptable limit in statistics. The differences are due to so much assumptions and approximations used in the classical approach in order to ease computations. Also, it can be seen, that deflection increased with increase in aspect ratio. The implication is that a square plate deflects less than other rectangular plates.

The coefficients of bending moment, β, obtained from the program developed for the bending analysis of CCCC plate, were compared with those of Ibearugbulem et al. (2013) in Table 4.33.

Table 4.33: Comparison of coefficients of bending moment 'β' obtained from the developed CCCC plate program with those of Ibearugbulem et al. (2013)

Aspect Ratio ; S = b/a	Present study $M_{xc}= \beta qa^2$; β	Ibearugbulem et al.(2013) $M_{xc} = \beta qa^2$; β_1	% difference $100(\beta- \beta_1)/ \beta_1$	Present study $M_{yc}= \beta_1 qa^2$; β_2	Ibearugbulem et al.(2013) $M_{yc} = \beta_1 qa^2$; β_3	% difference $100(\beta_2- \beta_3)/ \beta_3$
1.0	0.02765	0.02769	-0.14	0.02765	0.02769	-0.144
1.2	0.03517	0.03522	-0.14	0.02894	0.02899	-0.002
1.5	0.04270	0.04275	-0.12	0.02804	0.02808	-0.142
1.6	0.04441	0.04446	-0.11	0.02745	0.02748	-0.109
2.0	0.04877	0.04881	-0.08	0.02495	0.02497	-0.080
Aver. %diff.			-0.118			-0.095

For the moment at the center of the CCCC plate, the comparison indicates that the percentage difference between the values of the moments at the center of the plate in x-direction, M_{xc}, obtained from the program and those obtained by Ibearugbulem et al.(2013) are -0.14%, -0.14%, -0.12%, -0.11% and -0.08% for the same aspect ratios. And the average percentage difference is -0.118%. All these are less than 5% and are lower bound solutions to those compared with. This indicates that the results are close to those compared with. For the values of moment M_{yc}, considered in y- direction at the center of the plate, the percentage differences are -0.14%, -0.002%, -0.142%, -0.109%, and -0.080% respectively. Since the differences, are insignificant, the values can be said to be close to each other.

Table 4.34: Comparison of coefficients of shear force 'δ' obtained from the developed CCCC plate program with those of Ibearugbulem et al. (2014)

Aspect Ratio; S = b/a	Present study $V_{xmax} = \delta qa$; δ	Ibearugbulem et al(2014) $V_{xmax} = \delta qa$; δ_1	% difference $100(\delta - \delta_1)/\delta_1$	Present study $V_{ymax} = \delta_1 qa$; δ_2	Ibearugbulem et al(2014) $V_{ymax} = \delta_1 qa$; δ_3	% difference $100(\delta_2 - \delta_3)/\delta_3$
1.0	0.25521	0.2556	-0.15	0.25521	0.2556	-0.15
1.2	0.34924	0.3498	-0.16	0.20211	0.2024	-0.143
1.5	0.45212	0.4527	-0.13	0.13396	0.1341	-0.104
1.6	0.47699	0.4775	-0.11	0.11645	0.1166	-0.129
2.0	0.54444	0.5449	0.08	0.06806	0.0681	-0.059
Aver. %diff.			-0.126			-0.585

Table 4.34 contains the comparison between the results of the program for the determination of shear forces along the edges in both x- and y- directions. Comparing the values of the shear forces, V_x, in x-direction, obtained from the new program, with those obtained by Ibearugbulem et al. (2014), yielded percentage differences of -0.15%, -0.16%, -0.13%, -0.11% and -0.08% respectively for the same aspect ratios. This low percentage difference indicate the closeness of the values compared. For shear force, V_y, in y-direction, the percentage difference obtained are -0.15%, -0.143%, -0.104%, -0.129% and -0.059% respectively. This also shows the closeness of the values. The shear force in x-direction increased with increase in aspect ratio, while that in y-direction decreased with increase in aspect ratio.

Also, for bending analysis, the coefficient of edge moment, β, obtained from the computer program for CCCC plate, was compared in Table 4.35 with those obtained by Ibearugbulem et al. (2014).

Table 4.35: Comparison of coefficients of edge moment 'β' obtained from the developed CCCC plate program with those of Ibearugbulem et al. (2014)

Aspect Ratio ; S = b/a	Present study $M_{xe} = \beta qa^2$; β	Ibearugbulem et al.(2013) $M_{xe} = \beta qa^2$; β_1	% difference $100(\beta - \beta_1)/\beta_1$	Present study $M_{ye} = \beta_1 qa^2$; β_2	Ibearugbulem et al.(2013) $M_{ye} = \beta_1 qa^2$; β_3	% difference $100(\beta_2 - \beta_3)/\beta_3$
1.0	-0.0425	-0.0426	-0.23	-0.04252	-0.0426	-0.176
1.2	-0.0582	-0.0583	-0.17	-0.04042	-0.0405	-0.198
1.5	-0.0754	-0.0755	-0.13	-0.03349	-0.0335	-0.030
1.6	-0.0795	-0.0796	-0.10	-0.03105	-0.0311	-0.161
2.0	-0.0907	-0.0908	-0.11	-0.02269	-0.0227	-0.044
Aver. %diff.			-0.148			-0.123

In Table 4.35, the values of the edge moment in x- direction obtained from the present program were compared with those obtained by Ibearugbulem et al. (2014) for the same aspect ratios. The comparison gave the percentage differences as –0.23%, -0.17%, -0.13%, -0.10% and -0.11% respectively. These shows that these results are very close to the values they were compared with.

Also, the edge moment in y-direction, V_{ye}, indicates a maximum percentage difference of -0.198% at aspect ratio of 1.2. This value is less than 5%, and hence it is considered insignificant. It also indicate that the results obtained from the program, are lower bound to those obtained by Ibearugbulem.

Therefore, the program developed in this work, is adequate and useful as an easy alternative for analysis of rectangular CCCC plate.

The comparison of the results of analysis of SSSS and CCCC plates, show that deflection, moment and shear force of a simply supported edge are higher than those of a clamped edge which reflect a true practical situation.

Following the same analytical approach, for SSSS and CCCC plates above, the results of the other single panel plates are presented and discussed below.

Table 4.36: Comparison of coefficients of deflection 'α' obtained from the developed CSSS plate program with those of Ibearugbulem et al. (2014).

Aspect Ratio $S = b/a$	Present study $W_{max} = \alpha qa^4/D$; α	Ibearugbulem et al.(2014) $W_{max} = \alpha qa^4/D$; α_1	% difference $100(\alpha - \alpha_1)/\alpha_1$	Timoshenko et al.(1959) $W_{max} = \alpha qa^4/D$; α_2	% difference $100(\alpha - \alpha_2)/\alpha_2$
1.0	0.00282	0.00282	0.00	0.0028	0.00
1.2	0.00429	0.00429	0.00	0.0043	0.00
1.5	0.00646	0.00646	0.00	0.0064	0.936
1.6	0.00713	0.00713	0.00		
2.0	0.00937	0.00937	0.00	0.0093	0.753
Aver. %diff.			0.00		

The values in Table 4.36, were compared with results of pure bending analysis for CSSS plate obtained in existing literatures, based on exact and approximate methods. The comparison of the results of the deflection coefficients, obtained at the center of the plate, from the developed program with those obtained by Ibearugbulem et al. (2014), yielded percentage differences of 0.000%, 0.000%, 0.000%, 0.000% and 0.000% respectively for aspect ratios 1.0, 1.2, 1.5, 1.6, and 2.0. The average value of percentage difference is 0.000%. This is an indication that, these values agree closely. Also, these values obtained from the developed proram were compared with those values obtained by Timoshenko and Woinowsky-Krieger (1959), and this gave percentage differences are 0.000%, 0.000%, 0.938%, and 0.753% respectively for aspect ratios of 1.0, 1.2, 1.5 and 2. This further confirm that, the values obtained from the developed proram are very close to the exact values.

Also, for bending analysis, the coefficient of bending moment, β, obtained from the computer program for CSSS plate, was compared in Table 4.37 with those obtained by Ibearugbulem et al. (2014).

Table 4.37: Comparison of coefficients of bending moment 'β' obtained from the developed CSSS plate program with those of Ibearugbulem et al. (2014)

Aspect Ratio S = b/a	Present study $M_{xc}= \beta qa^2$ β	Ibearugbulem et al.(2014) $M_{xc} = \beta qa^2$ β_1	% difference $100(\beta - \beta_1)/\beta_1$	Present study $M_{yc}= \beta_1 qa^2$ β_2	Ibearugbulem et al.(2013) $M_{yc} = \beta_1 qa^2$ β_3	% difference $100(\beta_2 - \beta_3)/\beta_3$
1.0	0.03718	0.03718	0.00	0.04191	0.04191	0.00
1.2	0.05185	0.05185	0.00	0.04805	0.04805	0.00
1.5	0.07236	0.07236	0.00	0.05306	0.05306	0.00
1.6	0.07843	0.07842	0.013	0.05392	0.05392	0.00
2.0	0.09837	0.09837	0.00	0.05509	0.05509	0.00
Aver. %diff.			0.000			0.00

Furthermore, comparison was made between the values of the moment, M_{xc} , in x-direction, at the center of the plate with those values obtained from the work of Ibearugbulem et al. (2014), as indicated in Table 4.37, and the percentage differences are 0.000%, 0.000%, 0.000%, 0.013% and 0.000% for the same aspect ratios. The average percentage difference is 0.000%. This indicate that, results are absolutely close to those compared with. For moment, M_{yc}, in y-direction, the percentage differences are 0.000%, 0.000%, 0.000%, 0.000% and 0.000% for the same aspect ratio under consideration. Also, these show no difference at all.

Table 4.38: Comparison of coefficients of bending moment 'β' obtained from the developed CSSS plate program with those of Timoshenko & Woinowsky-Krieger (1959)

Aspect Ratio; S = b/a	Present study $M_{xc}= \beta qa^2$ β	Timoshen & Woinowsky-Krieger(1959) $M_{xc} = \beta qa^2$ β_1	% difference $100(\beta - \beta_1)/\beta_1$	Present study $M_{yc}= \beta_1 qa^2$ β_2	Timoshen & Woinowsky-Krieger(1959) $M_{yc} = \beta_1 qa^2$ β_3	% difference $100(\beta_2 - \beta_3)/\beta_3$
1.0	0.03718	0.034	8.824	0.04191	0.039	7.692
1.2	0.05185	0.049	6.122	0.04805	0.044	9.091
1.5	0.07236	0.069	4.348	0.05306	0.048	10.417
1.6	0.07843	-	-	0.05392	-	-
2.0	0.09837	0.094	4.255	0.05509	0.047	17.02
Aver. %diff.						

Comparing the values of moments, at the center of the plate with those values obtained by Timoshenko and Woinowsky-Krieger(1959) for aspect ratios 1.0, 1.2, 1.5 and 2, the percentage differences for moment, M_{xc} in x-direction, yielded 8.824%, 6.122%, 4.348%, and 4.255% respectively. These indicate that some of the values are above 5% but still within the acceptable range in statistics. And for moment, M_{yc}, in y-direction, the percentage differences for are 7.692%, 9.091%, 10.417% and 17.02% for aspect ratios of 1.0, 1.2, 1.5 and 2 respectively. They are upper bound to the exact values by Timoshenko and Woinowsky-Krieger but are also still within the acceptable statistic range.

Table 4.39: Comparison of coefficients of shear force 'δ' obtained from the developed CSSS plate program with those of Ibearugbulem et al. (2014)

Aspect Ratio S = b/a	Present study $V_{ymax} = \delta_1 qa$; δ_2	Ibearugbulem et al(2014) $V_{ymax} = \delta_1 qa$; δ_3	% difference $100(\delta_2 - \delta_3)/\delta_3$
1.0	0.33800	0.033800	0.00
1.2	0.29760	0.29760	0.00
1.5	0.22971	0.22971	0.00
1.6	0.20876	0.20875	0.005
2.0	0.14053	0.14053	0.00
Aver. %diff.			0.00095

Also, the coefficients of shear force, δ, obtained from the computer program for CSSS plate, were compared in Table 4.39 with those obtained by Ibearugbulem et al. (2014).

In Table 4.39, the percentage differences between the values of shear force in y-direction obtained from the present study with those obtained by Ibearugbulem et al. (2014) are insignificant. The maximum percentage difference occur at aspect ratio 1.6 and has a value 0.005.

Therefore, the conclusion is that, the present study Program is satisfactory and adequate as a quick way for analyzing rectangular CSSS plate.

Table 4.40: Comparison of coefficients of deflection 'α' obtained from the developed CSCS plate program with those of Ibearugbulem et al. (2014).

Aspect Ratio S = b/a	Present study $W_{max} = \alpha qa^4/D$; α	Ibearugbulem et al.(2014) $W_{max} = \alpha qa^4/D$; α_1	% difference $100(\alpha - \alpha_1)/\alpha_1$	Timoshenko et al.(1959) $W_{max} = \alpha qa^4/D$; α_2	% difference $100(\alpha - \alpha_2)/\alpha_2$
1.0	0.00199	0.00199	0.00	0.00192	3.64
1.2	0.00330	0.00330	0.00	0.00319	3.448
1.5	0.00551	0.00551	0.00	0.00531	3.766
1.6	0.00624	0.00624	0.00	0.00603	3.483
2.0	0.00886	0.00885	0.113	0.00844	4.976
Aver. %diff.					

To valid the results of this new computer program, obtained for CSCS plate, comparison was carried out between the values of the new study program with results of pure bending analysis for CSCS plate obtainable in some available literatures. In table 4.40, comparing the values of the deflection coefficients at the center of the plate obtained from the present program with those values obtained by Ibearugbulem et al. (2014), the percentage differences are 0.000% , 0.000%, 0.000%, 0.000% and 0.113% for aspect ratios 1.0, 1.2, 1.5, 1.6, and 2.0 respectively. These results agreed closely with those compared with. Also, in comparing these values with those obtained from the work of Timoshenko and Woinowsky-Krieger(1959) , the percentage difference for the same aspect ratios yielded 3.64%, 3.448%, 3.766%, 3.483%, and 4.979% respectively. These are all less than 5%, and are acceptable.

The coefficients of bending moment, β, obtained from the program developed for the bending analysis of a CSCS plate, were compared with those of Ibearugbulem et al. (20114) in Table 4.41.

Table 4.41: Comparison of coefficients of bending moment 'β' obtained from the developed CSCS plate program with those of Ibearugbulem et al. (2014)

Aspect Ratio S = b/a	Present study $M_{xc}= \beta qa^2$ β	Ibearugbulem et al.(2013) $M_{xc} = \beta qa^2$ β_1	% difference $100(\beta- \beta_1)/\beta_1$	Present study $M_{yc}= \beta_1 qa^2$ β_2	Ibearugbulem et al.(2013) $M_{yc} = \beta_1 qa^2$ β_3	% difference $100(\beta_2- \beta_3)/\beta_3$
1.0	0.02863	0.02310	23.94	0.03754	0.03755	-0.027
1.2	0.04268	0.03631	17.543	0.04618	0.04619	-0.022
1.5	0.06469	0.05787	11.785	0.05508	0.05508	0.00
1.6	0.07165	0.06486	10.469	0.05701	0.05701	0.00
2.0	0.09563	0.08945	6.909	0.06092	0.06091	0.016
Aver. %diff.			14.129			

For the moment at the center of CSCS plate, the comparison indicates that the percentage difference between the values of the moments, M_{xc}, at the center of the plate and those values obtained by Ibearugbulem et al.(2014), yielded 23.94%, 17.543%, 11.785%, 10.469% and 6.909% for the same aspect ratios. And the average percentage difference is 14.129%. Even though, this is above 5% and upper bound but the range is acceptable in statistics. This difference may be as a result of approximation used in the computation based on manual approach. For moment, M_{yc}, considered in the y-direction, the percentage differences are -0.027%, -0.022%, 0.000%, 0.000%, and 0.016% respectively. Since the differences, are insignificant, the values can be very close to each other.

Table 4.42: Comparison of coefficients of bending moment 'β' obtained from the developed CSCS plate program with those of Timoshenko and Woinowsky-Krieger (1959)

Aspect Ratio S = b/a	Present study $M_{xc}= \beta qa^2$ β	Timoshen & Woinowsky-Krieger(1959) $M_{xc} = \beta qa^2$ β_1	% difference $100(\beta- \beta_1)/\beta_1$	Present study $M_{yc}= \beta_1 qa^2$ β_2	Timoshen & Woinowsky-Krieger(1959) $M_{yc} = \beta qa^2$ β_3	% difference $100(\beta_2- \beta_3)/\beta_3$
1.0	0.02863	0.0244	17.336	0.03754	0.0332	13.072
1.2	0.04268	0.0376	22.819	0.04618	0.0400	15.450
1.5	0.06469	0.0585	10.581	0.05508	0.0460	19.739
1.6	0.07165	0.0650	10.231	0.05701	0.0469	21.557
2.0	0.09563	0.0869	10.046	0.06092	0.0474	28.523

Aver. %diff.						19.668

Also, when these same values were compared with those values obtained by Timoshenko and Woinowsky-Krieger(1959), the percentage differences for for moment, $M_{xc,}$ x-direction are 17.336%, 22.819%, 10.581%, 10.231% and 10.046% respectively, with an average percentage value of 14.20% . This average percentage is above 5%, nevertheless, it is acceptable in statistics. And for moment, M_{yc}, considered in y-direction, the percentage differences, yielded 3.072%, 15.450%, 19.739%, 21.557% and 28.523% respectively, with an average percentage difference value of 19.668%. Even though, the value is above 5%, but is acceptable in statistics as well.

Table 4.43: Comparison of coefficients of shear force 'δ' obtained from the developed CSCS plate program with those of Ibearugbulem et al. (2014)

Aspect Ratio S = b/a	Present study $V_{ymax} = \delta_1 qa$; δ_2	Ibearugbulem et al(2014) $V_{ymax} = \delta_1 qa$; δ_3	% difference $100(\delta_2 - \delta_3)/ \delta_3$
1.0	0.38175	0.38187	-0.031
1.2	0.36671	0.36679	-0.022
1.5	0.31362	0.31364	-0.638
1.6	0.29268	0.29269	-3.417
2.0	0.21251	0.21247	0.019
Aver. %diff.			

Also, in comparing the results of the shear forces, V_y, considered in y-diection, from the present study with those results obtained by Ibearugbulem et al.(2014) Table 4.43, for the same aspect ratios, the percentage differences are -0.031%, -0.022%, -0.638%, -3.417% and 0.019% respectively. These indicate that, the results are close to each other. It also, indicate that, as the aspect ratio increased shear force decreases.

The coefficients of the edge moment obtained from this new program was compared with those obtained by Ibearugbulem et al. (2014) in Table 4.44.

Table 4.44 Comparison of coefficients of edge moment 'β' obtained from the developed CSCS plate program with those of Ibearugbulem et al. (2014)

Aspect Ratio S = b/a	Present study $M_{ye} = \beta_1 qa^2$ β_2	Ibearugbulem et al.(2013) $M_{ye} = \beta_1 qa^2$ β_3	% difference $100(\beta_2 - \beta_3)/\beta_3$
1.0	-0.06363	-0.06365	-0.031
1.2	-0.07334	-0.07336	-0.027
1.5	-0.07841	-0.07841	0.00
1.6	-0.07805	-0.07805	0.00
2.0	-0.07084	-0.07082	0.028
Aver. %diff.			

In Table 4.44, comparison was made of the results of the edge moment, M_{ye}, considered in y- direction, obtained from the new program with those obtained by Ibearugbulem et al. (2014), the percentage differences are -0.031%, -0.027%, -0.000%, 0.000% and 0.028% respectively. These indicate that, the values are close to the values compared with. Also, it has shown from the results that, a simply supported edge does not carry moment.

Therefore, the conclusion that, the present study Program is adequate, and a quick means for obtaining the results of CSCS rectangular plate.

Table 4.45: Comparison of coefficients of deflection 'α' obtained from the developed CCSS plate program with those of Ibearugbulem et al. (2014).

Aspect Ratio S = b/a	Present study $W_{max} = \alpha qa^4/D$; α	Ibearugbulem et al.(2014) $W_{max} = \alpha qa^4/D$; α_1	% difference $100(\alpha - \alpha_1)/\alpha_1$
1.0	0.00210	0.00210	0.00
1.2	0.00290	0.00290	0.00
1.5	0.00386	0.00386	0.00
1.6	0.00411	0.00411	0.00
2.0	0.00486	0.00486	0.00
Aver. %diff.			

For CCSS plate, the coefficients of the deflection at the center of the plate, obtained from the new program in Table 4.45, were compared with results of pure bending analysis for CCSS plate, obtained by some scholars in order to validate the results. In comparing the values of the deflection coefficients at the center of the plate, obtained from the present program with those obtained by Ibearugbulem et al. (2014), the percentage difference for aspect ratios 1.0, 1.2, 1.5, 1.6, and 2.0 are 0.000%, 0.000%, 0.000%, 0.000% and 0.000% respectively. These values agree to the fact that, the results are close. This also, demonstrate the accuracy of the program.

Table 4.46: Comparison of coefficients of bending moment 'β' obtained from the developed CCSS plate program with those of Ibearugbulem et al. (2014)

Aspect Ratio; S = b/a	Present study $M_{xc}= \beta qa^2$ β	Ibearugbulem et al.(2013) $M_{xc} = \beta qa^2$ β_1	% difference $100(\beta- \beta_1)/\beta_1$	Present study $M_{yc}= \beta_1 qa^2$ β_2	Ibearugbulem et al.(2013) $M_{yc} = \beta_1 qa^2$ β_3	% difference $100(\beta_2- \beta_3)/\beta_3$
1.0	0.03277	0.03277	0.00	0.03277	0.03277	0.00
1.2	0.04203	0.04203	0.00	0.03459	0.03459	0.00
1.5	0.05247	0.05246	0.019	0.03446	0.03446	0.00
1.6	0.05511	0.05510	0.018	0.03407	0.03406	0.029
2.0	0.06266	0.06266	0.00	0.03206	0.03206	0.00
Aver. %diff.						

Furthermore, inTable 4.46, the comparison between the values of the moments; M_{xc} and M_{yc} at the center of the plate and those results obtained by Ibearugbulem et al. (2014), for the same aspect ratios, indicate that, the percentage differences are 0.000%, 0.000%, 0.019%, 0.018% and 0.000% for moment, $M_{xc,}$ considered in x-direction. The percentage differences are less than 5%, and indicate the absolute closeness of the values. For moment, M_{yc}, considered in y-direction, the percentage differences yielded 0.000%, 0.000%, 0.000%, 0.029%, and 0.000% respectively. These values show that, there are very close.

Table 4.47: Comparison of coefficients of shear force 'δ' obtained from the developed CCSS plate program with those of Ibearugbulem et al. (2014)

Aspect Ratio S = b/a	Present study $V_{xmax} = \delta qa$; δ	Ibearugbulem et al(2014) $V_{xmax} = \delta qa$; δ_1	% difference $100(\delta - \delta_1)/\delta_1$	Present study $V_{ymax} = \delta_1 qa$; δ_2	Ibearugbulem et al(2014) $V_{ymax} = \delta_1 qa$; δ_3	% difference $100(\delta_2 - \delta_3)/\delta_3$
1.0	0.25210	0.25209	0.004	0.25210	0.25209	0.004
1.2	0.34785	0.34784	0.003	0.20130	0.20130	0.00
1.5	0.46293	0.46292	0.002	0.13716	0.13716	0.00
1.6	0.49326	0.49324	0.004	0.12042	0.12042	0.00
2.0	0.58292	0.58290	0.003	0.07287	0.07286	0.014
Aver. %diff.						

Also, in comparing the results of the shear forces from the present study with those obtained by Ibearugbulem et al. (2014) as indicated in Table 4.47. For shear force, $V_{x,}$ considered in x-direction, the percentage differences are 0.004%, 0.003%, 0.002%, 0.004% and 0.003% respectively. An indication of how close the values are to each other. And considering shear forec, V_y, in y-direction, the percentages give 0.004%, 0.000%, 0.000%, 0.000% and 0.014% for the same aspect ratios under consideration. Also, this gives the indication that, the values are very close.

The coefficients of the edge moment, β, obtained from the computer program, were compared with those obtained by Ibearugbulem et al. (2014), as shown in Table 4.48.

Table 4.48: Comparison of coefficients of edge moment 'β' obtained from the developed CCSS plate program with those of Ibearugbulem et al. (2014)

Aspect Ratio S = b/a	Present study $M_{xe} = \beta qa^2$ β	Ibearugbulem et al.(2013) $M_{xe} = \beta qa^2$ β_1	% difference $100(\beta - \beta_1)/\beta_1$
1.0	-0.05042	-0.05042	0.00
1.2	-0.06957	-0.06957	0.00
1.5	-0.09259	-0.09258	0.011
1.6	-0.09865	-0.09865	0.00
2.0	-0.11658	-0.11658	0.00
Aver. %diff.			

In addition, Table 4.48 contains the comparison between the results of the edge moment, M_{xe}, considered in x- direction from this computer program with those obtained by Ibearugbulem et al. (2014), for the same aspect ratios under consideration. The percentage differences are 0.000%, 0.000%, 0.011%, 0.000% and 0.000% respectively. These indicate the closeness of the values to those compared with. Thus, the implication here is that, the present study Program, is a quick and useful program to obtain results for analysis of rectangular CCSS plate.

Table 4.49: Comparison of coefficients of deflection 'α' obtained from the developed CCCS plate program with those of Ibearugbulem et al. (2014).

Aspect Ratio S = b/a	Present study $W_{max} = \alpha q a^4/D$; α	Ibearugbulem et al.(2014) $W_{max} = \alpha q a^4/D$; α_1	% difference $100(\alpha - \alpha_1)/\alpha_1$
1.0	0.00161	0.00159	1.258
1.2	0.00243	0.00241	0.830
1.5	0.00355	0.00352	0.852
1.6	0.00387	0.00383	1.044
2.0	0.00482	0.00479	0.626
Aver. %diff.			0.922

In other to authenticate the results obtained from the new program, the coefficients of the deflection at the center of the plate obtained from this new program for CCCS plate, were compared with those coefficients obtained from Ibearugbulem et al. (2014), as indicated in Table 4.49. The comparison show that, the percentage difference for aspect ratios 1.0, 1.2, 1.5, 1.6, and 2.0 are 1.258% , 0.830%, 0.852%, 1.044% and 0.626% respectively, with an average value of 0.922%. This implies that the results of this study agree closely to those compared with.

Table 4.50: Comparison of coefficients of bending moment 'β' obtained from the developed CCCS plate program with those of Ibearugbulem et al. (2014)

Aspect Ratio S = b/a	Present study $M_{xc}= \beta qa^2$ β	Ibearugbulem et al.(2013) $M_{xc} = \beta qa^2$ β_1	% difference $100(\beta- \beta_1)/\beta_1$	Present study $M_{yc}= \beta_1 qa^2$ β_2	Ibearugbulem et al.(2013) $M_{yc} = \beta_1 qa^2$ β_3	% difference $100(\beta_2- \beta_3)/\beta_3$
1.0	0.02700	0.02673	1.010	0.03150	0.03119	0.994
1.2	0.03728	0.03690	1.036	0.03577	0.03540	1.045
1.5	0.05019	0.04971	0.966	0.03804	0.03767	0.982
1.6	0.05363	0.05314	0.922	0.03807	0.03772	0.928
2.0	0.06365	0.06318	0.744	0.03665	0.03638	0.742
Aver. %diff.			0.934			0.938

Furthermore, the percentage difference between the results of the moments, M_{xc} and M_{yc} at the center of the plate with values obtained by Ibearugbulem et al. (2014), for the same aspect ratios under consideration, presented in Table 4.50, indicate that, the percentage differences are 1.010%, 1.036%, 0.966%, 0.922% and 0.744% with an average value of 0.934 for moment, $M_{xc,}$ considered in x-direction. These values are less than 5%, and inplies that, the difference are insignificant. And considering moment, M_{yc}, in y-direction, the percentage differences are 0.994%, 1.045%, 0.982%, 0.928%, and 0.742% respectively, which implies that the results are close.

Moreso, the coefficient of the shear force, δ, obtained from this work were compared with those obtained by Ibearugbulem et al. (2014) as shown in Table 4.51.

Table 4.51: Comparison of coefficients of shear force 'δ' obtained from the developed CCCS plate program with those of Ibearugbulem et al. (2014).

Aspect Ratio; S = b/a	Present study $V_{xmax} = \delta qa$; δ	Ibearugbulem et al(2014) $V_{xmax} = \delta qa$; δ_1	% difference $100(\delta - \delta_1)/\delta_1$	Present study $V_{ymax} = \delta_1 qa$; δ_2	Ibearugbulem et al(2014) $V_{ymax} = \delta_1 qa$; δ_3	% difference $100(\delta_2 - \delta_3)/\delta_3$
1.0	0.19283	0.19095	0.985	0.30854	0.30552	0.988
1.2	0.29175	0.28875	1.039	0.27013	0.26736	1.036
1.5	0.42615	0.42206	0.969	0.20203	0.20009	0.970
1.6	0.46383	0.45958	0.925	0.18118	0.17952	0.925
2.0	0.57864	0.57440	0.738	0.11573	0.11488	0.740
Aver. %diff.			0.931			0.932

Based on the comparison, the percentage differences obtained for the moment, V_x, are 0.985%, 1.039%, 0.969%, 0.925% and 0.738%, with an average value of 0.931%. These indicate that, the percentage differences are negligible, and hence can be considered close to each other. While the percentage differences for moment, V_y, in y- direction, are 0.988%, 1.036%, 0.970%, 0.925%, and 0.740% respectively, this indicates the closeness of the results.

Table 4.52: Comparison of coefficients of edge moment 'β' obtained from the developed CCCS plate program with those of Ibearugbulem et al. (2014)

Aspect Ratio; S = b/a	Present study $M_{xe} = \beta qa^2$ β	Ibearugbulem et al.(2013) $M_{xe} = \beta qa^2$ β_1	% difference $100(\beta - \beta_1)/\beta_1$
1.0	-0.03857	-0.03819	0.995
1.2	-0.05835	-0.05775	1.039
1.5	-0.08523	-0.08441	0.971
1.6	-0.09277	-0.09192	0.925
2.0	-0.1157	-0.11488	0.714
Aver. %diff.			0.929

Moreover, in Table 4.52, comparison was made between the results of the edge moments in x- direction, obtained from the present program with those obtained by Ibearugbulem et al. (2014). The results indicate that the percentage differences are 0.995%, 1.039%, 0.971%, 0.925% and 0.714% respectively with an average of 0.929%. The implication is that, the results are close and satisfactory. The edge moment increased with increase in aspect ratio in x-direction. All the moments are negative meaning they act in opposite direction to the assumed direction.

Thus, the implication is that, the present study Program, is a reliable and faster approach for analysis of rectangular CCCS plate. Also, the program, gives flexibility in usage by the user in terms of varying parameters as the practical problems present themselves.

Table 4.53: Comparison of coefficients of deflection 'α' obtained from the developed SSFS plate program with those of Ibearugbulem et al. (2014).

Aspect Ratio; S = b/a	Present study $W_{max} = \alpha qa^4/D$; α	Ibearugbulem et al.(2014) $W_{max} = \alpha qa^4/D$; α_1	% difference $100(\alpha - \alpha_1)/\alpha_1$	Timoshenko et al.(1959) $W_{max} = \alpha qa^4/D$; α_2	% difference $100(\alpha - \alpha_2)/\alpha_2$
1.0	0.01105	0.01282	-13.807	0.01286	-14.075
1.2	0.01250	0.01403	-10.905	0.01384	-9.682
1.5	0.01393	0.01511	-7.809	0.01462	-4.720
1.6	0.01428	0.01535	-6.971	-	-
2.0	0.01524	0.01601	-4.809	0.01507	1.128
Aver. %diff.			-8.860		-6.837

In Table 4.53, comparison of the deflection coefficients, α, at the free edge of the plate, obtained from the present study with those obtained by Ibearugbulem et al. (2014), yielded percentage difference is -13.807%,-10.905%, -7.809%, -6.971% and -4.809% respectively for aspect ratios 1.0, 1.2, 1.5, 1.6, and 2.0. The average percentage difference was -8.860%. These values are lower bound to those compared with, and the average is above 5%. However, the results are within the acceptable limit in statistics.

Comparing the same values with those of Timoshenko and Woinowsky-Krieger (1959), gave percentage difference of -14.075%, -9.682%, -4.720%, and 1.128% respectively for aspect ratios 1.0, 1.2, 1.5, and 2.0, and with an average percentage difference of -6.837%. This value is sligtly above 5%, but is within acceptable limit in statistics.

Table 4.54: Comparison of coefficients of bending moment 'β' obtained from the developed SSFS plate program with those of Ibearugbulem et al. (2014)

Aspect Ratio; S = b/a	Present study $M_{xc}= \beta qa^2$ β	Ibearugbulem et al.(2013) $M_{xc} = \beta qa^2$ β_1	% difference $100(\beta- \beta_1)/\beta_1$	Present study $M_{yc}= \beta_1 qa^2$ β_2	Ibearugbulem et al.(2013) $M_{yc} = \beta_1 qa^2$ β_3	% difference $100(\beta_2- \beta_3)/\beta_3$
1.0	0.08007	0.0928	-13.718	0.04293	0.0369	16.341
1.2	0.08843	0.1015	-12.877	0.04138	0.0403	2.680
1.5	0.09659	0.1093	-11.629	0.03957	0.0434	-8.825
1.6	0.09853	0.1111	-11.314	0.03910	0.0441	-11.338
2.0	0.10397	0.1159	-10.293	0.03771	0.0460	-18.022
Aver. %diff.			-11.966			-3.833

Furthermore, the coefficients of bending moments, β, obtained from the program developed for bending analysis of SSFS plate, were compared with those of Ibearugbulem et al. (2014) in Table 4.54. The comparison indicates that the percentage difference between values of the moments, M_{xc}, considered in x-direction, at the center of the plate and those obtained by Ibearugbulem et al. (2014), are -13.718%, -12.877%, -11.629%, -11.314% and -10.293%, for the same aspect ratio. And the average percentage difference is -11.966%. All values are above than 5%, and are lower bound solutions to those compared with. This indicates that, the results even though above 5%, are within acceptable range in statistics. For the values of moment, M_{yc}, considered in y-direction at the center of centage of the plate, the percentage differences are 16.341%, 2.680%, -8.825%, -11.338%, and -18.022% respectively. These values are acceptable even though there are above 5%. These differences arise from the fact that, so many assumptions may have been used in the manual computation of results, to simplify the process, since plates with free edge do not follow exactly, the derivations as the plates with no free edge.

Table 4.55: Comparison of coefficients of bending moment 'β' obtained from the developed SSFS plate program with those of Timoshenko and Woinowsky-Krieger (1959).

Aspect Ratio; S = b/a	Present study $M_{xc}= \beta qa^2$ β	Timoshen & Woinowsky-Krieger(1959) $M_{xc} = \beta qa^2$ β_1	% difference $100(\beta- \beta_1)/\beta_1$	Present study $M_{yc}= \beta_1 qa^2$ β_2	Timoshen & Woinowsky-Krieger(1959) $M_{yc} = \beta qa^2$ β_3	% difference $100(\beta_2- \beta_3)/\beta_3$
1.0	0.08007	0.080	0.00	0.04293	0.039	10.077
1.2	0.08843	0.090	-1.778	0.04138	0.041	0.927
1.5	0.09659	0.101	-4.366	0.03957	0.042	-5.785
1.6	0.09853	-	-	0.03910	-	-
2.0	0.10397	0.0.113	-7.799	0.03771	0.041	-8.024
Aver. %diff.			-3.486			-0.701

To further validate the results of this computer program, these same values were compared with those obtained by Timoshenko and Woinowsky-Krieger(1959), in Table 4.55, and the percentage differences for moment, $M_{xc,}$ considered in x-direction are 0.000%, -1.778%, -4.366% and -7.799% respectively for aspect ratios 1.0, 1.2, 1.5, and 2.0. And the average percentage diference of -3.486%. These values are less than 5%, and indicate that the values are closer than those of Ibearugbulem et al. Also, considering moment, M_{yc}, in y-direction, the perecentage differences are 10.077%, 0.927%, -5.785% and -8.024% respectively, with an average percentage difference of -0.701%. This indicates that the values are very close to each other. The insignicant differences affirm the fact that these values are adequate.

Table 4.56: Comparison of coefficients of shear force 'δ' obtained from the developed SSFS plate program with those of Ibearugbulem et al. (2014)

Aspect Ratio; S = b/a	Present study $V_{xmax} = \delta qa$; δ	Ibearugbulem et al(2014) $V_{xmax} = \delta qa$; δ_1	% difference $100(\delta- \delta_1)/\delta_1$
1.0	0.40839	0.4731	-13.678
1.2	0.42293	0.5175	-18.274
1.5	0.43570	0.5574	-21.834
1.6	0.43857	0.5664	-22.569
2.0	0.44626	0.5907	-24.452

Aver. %diff.			-20.16

Also, in table 4.56, comparing the resuts of the shear forces, V_x, considered in x-direction, obtained from the present program with those obtained by Ibearugbulem et al. (2014), the percentage differences are -13.678%, -18.274%, -21.834%, -22.569% and -24.452% respectively, with an average percentage difference of -20.16% . This value is above 5%, but acceptable in statistics.

Therefore, the present study Program, can be considered adequate, and as a quick alternative for analysis of rectangular SSFS plate.

Table 4.57: Comparison of coefficients of deflection 'α' obtained from the developed SCFS plate program with those of Ibearugbulem et al. (2014).

Aspect Ratio; S = b/a	Present study $W_{max} = \alpha qa^4/D$ α	Ibearugbulem et al.(2014) $W_{max} = \alpha qa^4/D$ α_1	% difference $100(\alpha - \alpha_1)/\alpha_1$
1.0	0.00547	0.00672	-18.601
1.2	0.00586	0.00680	-13.824
1.5	0.00621	0.00685	-9.343
1.6	0.00628	0.00686	-8.455
2.0	0.00650	0.00688	-5.523
Aver. %diff.			-11.14

Table 4.57, contains the comparison between the results obtained from this new program with those results obtained for SCFS plate by existing research work. The comparison indicates that, the percentage difference between the values of the deflection coefficients at the free edge of the plate obtained from the new study program with those obtained by Ibearugbulem et al. (2014), are -18.601%, -13.824%, -9.343%, -8.455% and -5.523% respectively for aspect ratios 1.0, 1.2, 1.5, 1.6, and 2.0. And the average percentage difference of -11.149%. This is lower bound and acceptable.

Furthermore, the coefficients of bending moment, β, obtained from the program developed for bending analysis of a SCFS plate were compared with those of Ibearugbulem et al. (2014), in Table 4.58.

Table 4.58: Comparison of coefficients of bending moment 'β' obtained from the developed SCFS plate program with those of Ibearugbulem et al. (2014)

Aspect Ratio; S = b/a	Present study $M_{xc} = \beta qa^2$ β	Ibearugbulem et al.(2013) $M_{xc} = \beta qa^2$ β_1	% difference $100(\beta - \beta_1)/\beta_1$
1.0	0.04877	0.0599	-18.581
1.2	0.05123	0.0606	-15.462
1.5	0.05339	0.0610	-12.475
1.6	0.05387	0.0611	-11.833
2.0	0.05517	0.0613	-10.000
Aver. %diff.			-13.670

For the moments, M_{xc}, considered at the center of the plate, the comparison indicates that the percentage difference between the results of this computer program and those obtained by Ibearugbulem et al.(2014), are -18.581%, -15.462%, -12.475%, -11.833% and -10.000%. And the average percentage value of -13.670%. These are higher than 5%, and lower bound to those compared with, but acceptable in statistics.

Table 4.59: Comparison of coefficients of shear force 'δ' obtained from the developed SCFS plate program with those of Ibearugbulem et al. (2014).

Aspect Ratio S = b/a;	Present study $V_{xmax} = \delta qa$; δ	Ibearugbulem et al(2014) $V_{xmax} = \delta qa$; δ_1	% difference $100(\delta - \delta_1)/\delta_1$
1.0	0.45688	0.5611	-18.574
1.2	0.48935	0.5677	-13.801
1.5	0.51834	0.5718	-9.349
1.6	0.52489	0.5725	-8.316
2.0	0.54253	0.5740	-5.483
Aver. %diff.			-11.483

Also, comparing the values of the shear forces, V_x, considered in x-direction, obtained from the present work with those obtained by Ibearugbulem et al.(2014) for the same aspect ratios, the percentage differences are -18.574%, -13.801%, -9.349%, -8.316% and -5.483% respectively. These values are lower bound to those compared with, and are satisfactory.

Table 4.60: Comparison of coefficients of edge moment 'β' obtained from the developed SCFS plate program with those of Ibearugbulem et al. (2014)

Aspect Ratio; S = b/a	Present study $M_{xe} = \beta qa^2$ β	Ibearugbulem et al.(2013) $M_{xe} = \beta qa^2$ β_1	% difference $100(\beta- \beta_1)/ \beta_1$
1.0	-0.09138	-0.1122	-18.556
1.2	-0.09787	-0.1135	-13.771
1.5	-0.10367	-0.1144	-9.379
1.6	-0.10498	-0.1145	-8.314
2.0	-0.10851	-0.1148	-5.479
Aver. %diff.			-11.100

In addition, comparing the values of the edge moment, M_{xe}, considered in y-direction, obtained from the present study with those obtained by Ibearugbulem et al. (2014), for the same aspect ratios, the percentage differences are -18.556%, -13.771%, -9.379%, -8.314% and -5.479% respectively. This percentage differences are considered acceptable. Hence, the present study Program is a reliable and adequate program, for quicker analysis of rectangular SCFS plate.

Table 4.61: Comparison of coefficients of deflection 'α' obtained from the developed CSFS plate program with those of Ibearugbulem et al. (2014).

Aspect Ratio S = b/a	Present study $W_{max} = \alpha qa^4/D$; α	Ibearugbulem et al.(2014) $W_{max} = \alpha qa^4/D$; α_1	% difference $100(\alpha- \alpha_1)/ \alpha_1$	Timoshenko et al.(1959) $W_{max} = \alpha qa^4/D$; α_2	% difference $100(\alpha- \alpha_2)/ \alpha_2$
1.0	0.00884	0.0101	-12.475		-
1.2	0.01094	0.0122	-10.328		
1.5	0.01301	0.0141	-7.730	0.0141	-7.773
1.6	0.01350	0.0145	-6.897		
2.0	0.01482	0.0156	-5.000	0.0150	-1.2
Aver. %diff.			-8.486		

The coefficients of deflection, α, obtained from the developed CSFS plate program were compared with those obtained by Ibearugbulem et al. (2014) in Table 4.61. The comparison shows that, the percentage differences are -12.475%, -10.328%, -7.730%, -6.897% and -5.000% respectively for aspect ratios 1.0, 1.2, 1.5, 1.6, and 2.0. These values are lower bound to those compared with and are considered satisfactory.

Table 4.62: Comparison of coefficients of bending moment 'β' obtained from the developed CSFS plate program with those of Ibearugbulem et al.(2014).

Aspect Ratio; S = b/a	Present study $M_{xc} = \beta qa^2$ β	Ibearugbulem et al.(2013) $M_{xc} = \beta qa^2$ β_1	% difference $100(\beta - \beta_1)/\beta_1$
1.0	0.06167	0.0702	-12.151
1.2	0.07352	0.0851	-13.608
1.5	0.08478	0.0986	-14.016
1.6	0.0873	0.1015	-13.016
2.0	0.09417	0.1089	-13.562
Aver. %diff.			-13.558

Furthermore, Table 4.62, contains comparison between the cofficients of bending moment, β, considered at the center of the plate, and those obtained by Ibearugbulem et al. (2014). The comparison show that, for moments, $M_{xc,}$ the percentage differences are -12.151%, -13.608%, -14.016%, -13.016% and -13.526%, with an average value of -13.458%. These are are above than 5%, and lower bound but acceptable in statistics.

Therefore, the present study Program, is adequate and an easy way of analyzing CSFS rectangular plate.

Table 4.63: Comparison of coefficients of deflection 'α' obtained from the developed CCFS plate program with those of Ibearugbulem et al. (2014).

Aspect Ratio S = b/a	Present study $W_{max} = \alpha qa^4/D$; α	Ibearugbulem et al.(2014) $W_{max} = \alpha qa^4/D$; α_1	% difference $100(\alpha - \alpha_1)/\alpha_1$
1.0	0.00485	0.0053	-8.491
1.2	0.00548	0.0058	-5.517
1.5	0.00601	0.0063	-4.603
1.6	0.00612	0.0064	-4.375
2.0	0.00641	0.0066	-2.879
Aver. %diff.			-5.173

Also, the results of shear force coefficients, α, at the edge of the plate obtained from the present study were compared with results of pure bending analysis for CCFS plate obtained by research work of Ibearugbulem et al. (2014). From the comparison as indicated in Table 4.63, the percentage differences recorded between the values are -8.491%, -5.517%, -4.603%, -4.375% and -2.879% respectively, with an average value of -5.173%, for the same aspect ratios. These values are lower bound to those compared with, and are satisfactory.

Table 4.64: Comparison of coefficients of bending moment 'β' obtained from the developed CCFS plate program with those of Ibearugbulem et al. (2014)

Aspect Ratio S = b/a	Present study $M_{xc} = \beta qa^2$ β	Ibearugbulem et al.(2013) $M_{xc} = \beta qa^2$ β_1	% difference $100(\beta - \beta_1)/\beta_1$	Present study $M_{yc} = \beta_1 qa^2$ β_2	Ibearugbulem et al.(2013) $M_{yc} = \beta_1 qa^2$ β_3	% difference $100(\beta_2 - \beta_3)/\beta_3$
1.0	0.04127	0.0448	-7.879	0.02451	0.0205	19.561
1.2	0.04524	0.0497	-8.974	0.02309	0.0228	1.272
1.5	0.04834	0.0534	-9.476	0.02120	0.0245	-13.469
1.6	0.04903	0.0542	-9.539	0.02069	0.0249	-16.908
2.0	0.05063	0.0561	-9.750	0.0192	0.0257	-25.292
Aver. %diff.			-9.124			-6.967

Furthermore, in table 4.64, the percentage difference between results of the moments, M_{xc}, and those values obtained by Ibearugbulem et al.(2014), at the center of the plate, for the same aspect ratios, shows that, the percentage differences are -7.879%, -8.974%, -9.476%, -9.539% and -9.750%, with an average value of -9.124%. These values are slightly above 5%, and acceptable in statistics. And for moment, M_{yc}, the percentage differences are 19.561%, 1.272%, -13.469%, -16.908%, and -25.292% respectively, with an average value of -6.967%. These are acceptable as well.

Table 4.65: Comparison of coefficients of shear force 'δ' obtained from the developed CCFS plate program with those of Ibearugbulem et al. (2014)

Aspect Ratio S = b/a	Present study $V_{xmax} = \delta qa$ δ	Ibearugbulem et al(2014) $V_{xmax} = \delta qa$ δ_1	% difference $100(\delta - \delta_1)/\delta_1$
1.0	0.37269	0.4043	-7.818
1.2	0.42100	0.4485	-6.132
1.5	0.46183	0.4826	-4.304
1.6	0.47058	0.4895	-3.865
2.0	0.49308	0.5062	-2.592
Aver. %diff.			-4.942

Also, in table 4.65, comparing the results of the shear forces, V_x, obtained from the new program with those obtained by Ibearugbulem et al. (2014), shows that, the percentage differences are -7.818%, -6.132%, -4.304%, -3.865% and -2.592% respectively. These values are below 5%. The values are lower bound to those compared with. The maximum difference occur at an aspect ratio of 1.0, and started dcreasing as the aspect ratio increase. The results indicate insignificant differences with those compared with and are satisfactory.

Table 4.66: Comparison of coefficients of edge moment 'β' obtained from the developed CCFS plate program with those of Ibearugbulem et al. (2014)

Aspect Ratio S = b/a	Present study $M_{xe} = \beta qa^2$ β	Ibearugbulem et al.(2013) $M_{xe} = \beta qa^2$ β_1	% difference $100(\beta - \beta_1)/\beta_1$
1.0	-0.07454	-0.0809	-7.862
1.2	-0.0842	-0.0897	-6.132
1.5	-0.09237	-0.0965	-4.280
1.6	-0.09412	-0.0979	-3.861
2.0	-0.09862	-0.1012	-2.549
Aver. %diff.			-4.937

In addition, Table 4.66, contains the comparison between the results of the edge moment, $M_{xe,}$ considered in x- direction, obtained from the present study with those obtained by Ibearugbulem et al. (2014). The percentage differences obtained from the comparison are -7.862%, -6.132%, -4.280%, -3.861% and -2.549% respectively.And an average percentage difference of -4.937%. This is below 5%. This implies that, the results are close and satisfactory. Therefore, we can say that, the present study program, is adequate and quicker for analysis of rectangular CCFS plate.

Table 4.67: Comparison of coefficients of deflection 'α' obtained from the developed SCFC plate program with those of Ibearugbulem et al. (2014).

Aspect Ratio S = b/a	Present study $W_{max} = \alpha qa^4/D$ α	Ibearugbulem et al.(2014) $W_{max} = \alpha qa^4/D$ α_1	% difference $100(\alpha- \alpha_1)/ \alpha_1$
1.0	0.00305	0.0032	-4.688
1.2	0.00317	0.0033	-3.939
1.5	0.00327	0.0033	-0.909
1.6	0.00329	0.0034	-3.235
2.0	0.00335	0.0034	-1.471
Aver. %diff.			-2.848

In table 4.67, comparison of the deflection coefficients, α, at the edge of the plate, obtained from the present study with those obtained by Ibearugbulem et al. (2014), yielded percentage differences of -4.688% , -3.939%, -0.909%, -3.235% and -1.471% respectively, with an average value of -2.848%. These values indicate that they agree closely with each other. The results also indicate that, as the aspect ratio increases deflection also increased.

Table 4.68: Comparison of coefficients of bending moment 'β' obtained from the developed SCFC plate program with those of Ibearugbulem et al. (2014)

Aspect Ratio S = b/a	Present study $M_{xc}= \beta qa^2$ β	Ibearugbulem et al.(2013) $M_{xc} = \beta qa^2$ β_1	% difference $100(\beta- \beta_1)/ \beta_1$
1.0	0.03571	0.0374	-4.637
1.2	0.036524	0.0383	-4.615
1.5	0.03720	0.0390	-4.615
1.6	0.03734	0.0392	-4.745
2.0	0.03772	0.0396	-4.747
Aver. %diff.			-4.653

Furthermore, in table 4.68, the percentage difference between the results of the moments, M_{xc}, at the center of the plate with values obtained by Ibearugbulem et al.(2014), shows that, the percentage differences are -4.637%,-4.615%, -4.615%, -4.745% and -4.747%, with an average value of -4.653%. These percentages are all below 5%, and lower bound and are satisfactory. The deflection increased as aspect ratio increases.

Table 4.69: Comparison of coefficients of shear force 'δ' obtained from the developed program for SCFC plate with those of Ibearugbulem et al. (2014).

Aspect Ratio S = b/a	Present study $V_{xmax} = \delta qa$ δ	Ibearugbulem et al(2014) $V_{xmax} = \delta qa$ δ_1	% difference $100(\delta- \delta_1)/ \delta_1$
1.0	0.40781	0.4267	-4.427
1.2	0.42340	0.4373	-3.179
1.5	0.43654	0.4459	-2.099
1.6	0.43941	0.4477	-1.852
2.0	0.44700	0.4524	-1.194
Aver. %diff.			-2.550

Also, comparing the results of the shear forces, V_x, obtained from the present study with those obtained by Ibearugbulem et al. (2014) in Table 4.69. The percentage differences are -4.427%, -3.179%, -2.099%, -1.852% and -1.194% respectively. And an average average percentage difference of -2.550%. All the values are below 5%, and are lower bound to those compared with. Hence, are considered adequate.

Table 4.70: Comparison of coefficients of edge moment ‘β’ obtained from the developed program SCFC plate with those of Ibearugbulem et al. (2014)

Aspect Ratio S = b/a	Present study $M_{xe} = \beta qa^2$ β	Ibearugbulem et al.(2013) $M_{xe} = \beta qa^2$ β_1	% difference $100(\beta - \beta_1)/\beta_1$
1.0	-0.06797	-0.0711	-4.402
1.2	-0.07057	-0.0729	-3.196
1.5	-0.07276	-0.0743	-2.073
1.6	-0.07324	-0.0746	-1.823
2.0	-0.07450	-0.0754	-1.194
Aver. %diff.			-2.538

In addition, comparing coefficients of the edge moment, β, in x- direction, obtained from the present study with those obtained by Ibearugbulem et al. (2014), as indicated in Table 4.70. The comparison gave the percentage differences as -4.402%, -3.196%, -2.073%, -1.823% and -1.194% respectively, with an average percentage difference of -2.538%. This is below 5%, and considered satisfactory.

Therefore, the results of the present study program, are reliable and adequate, and offer a quicker way of obtaining results of rectangular SCFC plate.

Table 4.71: Comparison of coefficients of deflection 'α' obtained from the developed CCFC plate program with those of Ibearugbulem et al. (2014).

Aspect Ratio S = b/a	Present study $W_{max} = \alpha qa^4/D$ α	Ibearugbulem et al.(2014) $W_{max} = \alpha qa^4/D$ α_1	% difference $100(\alpha - \alpha_1)/\alpha_1$	Timoshenko et al.(1959) $W_{max} = \alpha qa^4/D$ α_2	% difference $100(\alpha - \alpha_2)/\alpha_2$
1.0	0.00286	0.00299	-4.348	0.00333	-14.11
1.2	0.00306	0.00316	-3.165	-	-
1.5	0.00321	0.00329	-2.432	0.00335	-4.179
1.6	0.00325	0.00331	-1.813	-	-
2.0	0.00333	0.00337	-1.187	-	-
Aver. %diff.			-2.589		

Comparison was carried out in Table 4.71, between the results obtained from the developed CCFC plate program and those results of pure bending analysis for CCFC plate obtained by some researchers from existing literatures.

Comparing the values of the deflection coefficient at the free edge of the plate, obtained from the present study with those obtained by Ibearugbulem et al. (2014), the percentage differences are -4.348%, -3.165%, -2.432%, -1.813% and -1.187% respectively, for aspect ratios 1.0, 1.2, 1.5, 1.6, and 2.0. And an average percentage difference of -2.589%. This indicates that the results agree closely. The reason for some slight differeces in these results may be as a resut of some approximations and rigorous manually computations.

Table 4.72: Comparison of coefficients of bending moment 'β' obtained from the developed CCFC plate program with those of Ibearugbulem et al. (2014)

Aspect Ratio S = b/a	Present study $M_{xc}= \beta qa^2$ β	Ibearugbulem et al.(2013) $M_{xc} = \beta qa^2$ β_1	% difference $100(\beta- \beta_1)/ \beta_1$	Timoshenko et al.(1959) $W_{max} = \alpha qa^4/D$ α_2	% difference $100(\beta- \beta_1)/ \beta_1$
1.0	0.03165	0.033150	-4.525	0.0317	0.00
1.2	0.03310	0.035052	-5.569	-	-
1.5	0.03412	0.036408	-6.284	0.0402	-15.121
1.6	0.03432	0.036670	-6.409	-	-
2.0	0.03477	0.037298	-6.778	-	-
Aver. %diff.			-5.913		

Furthermore, in Table 4.72, the percentage difference between the results of the moment, M_{xc}, in x-direction, at the center of the plate with values obtained by Ibearugbulem et al. (2014), indicate that the percentage differences are -4.525%, -5.569%, -6.284%, -6.409% and -6.778%, with an average value of -5.913%. These difference are around the neighbourhood of 5%, and are lower bound to those compared with. This indicates that the values are close. And comparing the same values with those obtained by Timoshenko and Woinowsky-Krieger (1959), for moment, M_{xc}, the percentage differences for aspect ratios 1 and 1.5 are 0.000% and -15.121% respectively. These indicate that the values for 1 is absolutely the same, while that for 1.5 is lower bound and acceptable.

Table 4.73: Comparison of coefficients of shear force 'δ' obtained from the developed CCFC plate program with those of Ibearugbulem et al. (2014).

Aspect Ratio S = b/a	Present study $V_{xmax} = \delta qa$ δ	Ibearugbulem et al(2014) $V_{xmax} = \delta qa$ δ_1	% difference $100(\delta - \delta_1)/\delta_1$
1.0	0.35146	0.368178	-4.541
1.2	0.37623	0.389293	-3.356
1.5	0.39533	0.404357	-2.232
1.6	0.39923	0.407269	-1.974
2.0	0.40898	0.414237	-1.269
Aver. %diff.			-2.674

Also, in Table 4.73, comparing the results of the shear forces, V_x, considered in x-direction, obtained from the present study with those obtained by Ibearugbulem et al. (2014), the percentage differences are -4.541%, -3.356%, -2.232%, -1.974% and -1.269% respectively, with an average of -2.674%. These values are less than 5%, and indicate the closeness of the values.

Table 4.74: Comparison of coefficients of edge moment 'β' obtained from the developed CCFC plate program with those of Ibearugbulem et al. (2014).

Aspect Ratio S = b/a	Present study $M_{xe} = \beta qa^2$ β	Ibearugbulem et al.(2013) $M_{xe} = \beta qa^2$ β_1	% difference $100(\beta - \beta_1)/\beta_1$
1.0	-0.05858	-0.06136	-4.531
1.2	-0.06270	-0.06488	-3.360
1.5	-0.06589	-0.06739	-2.226
1.6	-0.06654	-0.06788	-1.974
2.0	-0.06816	-0.06904	-1.275
Aver. %diff.			-2.673

In addition, in Table 4.74, comparing the values of the edge moment in x-direction, obtained from the present study with those obtained by Ibearugbulem et al. (2014), the percentage differences are -4.531%, -3.360%, -2.226%, -1.974% and -1.275% respectively, with an average of -2.673%. This indicates that the percentage differences are less than 5% and are close to the compared values.

Therefore, the implication is that, the present study program, is adequate and quicker to analyze rectangular CCFC plate.

It is worth noting that, the manual computations, based on both classical and energy approach, make used of approximations to various decimal places for various plates. This may have account for some of the little differences in results, even though the differences are within acceptable limits. We also obersved that, in one way or the other, as the aspect ratio increased, there is some convergences or divergence of the values. But within the acceptable practical limit of aspect ratio of 1<s<2.5, the results are adequate and satisfactory.

4.2.2 Discussions of the Results of Buckling/Stability Analysis

In other to validate the results, obtained from this program for stability analysis, Tables 4.75 – 4.80 show the comparison of these results with those obtained from available literatures, and the discussions are as follows.

Table 4.75: Comparison of coefficients of critical buckling load 'n' obtained from the developed program with those of Ibearugbulem et al. (2014)

Aspect Ratio; S = b/a	SSSS Plate			CCCC Plate		
	Program $N_x = n_{2x}\frac{D}{b^2}$; n	Ibearugbulem et al.(2014) n_1	% difference $100(n- n_1)/ n_1$	Program $N_x = n_{2x}\frac{D}{b^2}$; n_2	Ibearugbulem et al.(2014) n_3	% difference $100(n_2- n_3)/ n_3$
1.0	4.003	4.003	0.00	10.943	11.010	-0.61
1.2	4.137	4.138	-0.024	11.515	11.587	-0.62
1.5	4.698	4.698	0.00	13.898	13.989	-0.65
1.6	4.955	4.955	0.00	14.988	15.088	-0.66
2.0	6.256	6.256	0.00	20.518	20.661	-0.69
Aver. %diff.			-0.005			-0.646

To validate the results obtained from the program for SSSS and CCCC plates, Ttable 4.75 show the comparison of these results with those obtained by Ibearugbulem et al. (2014).

For SSSS plate, for aspect ratio 1.0, 1.2, 1.5, 1.6, and 2.0, the percentage differences have a maximum value of -0.024% at aspect ratio 1.2, while other are 0.00%, with average percentage differences of -0.005%. These values indicate an insignificant difference and thus agree very well.

And for CCCC plate, in the same Table 4.75, were compared with the same authors, and the percentage differences are -0.609%, -0.621%, -0.651%, -0.665% and -0.692%, with an average value of -0.647%. Also, are very close and show only insignificant difference. Hence, the implication is that this study program is accurate and adequate for analyzing SSSS and CCCC rectangular plates.

Following the same analytical procedure, the rest of the other plates' results are compared and discussed as follows:

Table 4.76: Comparison of coefficients of critical buckling load 'n' obtained from the developed program with those of Ibearugbulem et al. (2014)

Aspect Ratio; S = b/a	CSSS Plate			CSCS Plate		
	Program $N_x = n_{2x}\frac{D}{b^2}$; n	Ibearugbulem et al.(2014) n_1	% difference $100(n-n_1)/n_1$	Program $N_x = n_{2x}\frac{D}{b^2}$; n_2	Ibearugbulem et al.(2014) n_3	% difference $100(n_2-n_3)/n_3$
1.0	5.756	5.755	0.017	8.606	8.619	-0.151
1.2	5.447	5.447	0.00	7.466	7.478	-0.160
1.5	5.646	5.646	0.00	6.984	6.996	-0.172
1.6	5.824	5.824	0.00	7.016	7.028	-0.171
2.0	6.922	6.921	0.014	7.730	7.745	-0.194
Aver. %diff.			0.006			-0.169

In Table 4.76, the values for critical buckling load obtained from the program for CSSS plate were compared with those obtained by Ibearugbulem et al. (2014), the percentage differences are 0.017%, 0.000%, 0.000%, 0.000% and 0.014% respectively for aspect ratio 1.0, 1.2, 1.5, 1.6, and 2.0. The average percentage differences is 0.006%. These values show an insignificant difference and thus agree very absolutely.

And for CSCS plate in Table 4.76, the values of critical buckling load obtained from the program, were compared with the same authors. And the percentage differences are -0.151%, -0.160%, -0.172%, -0.171% and -0.194%, with an average value of -0.169%. Also, these values are very close and indicate only slight difference which are insignificant. Hence, the implication is that, this study program is accurate and useful approach for analyzing CSSS and CSCS rectangular plates.

Table 4.77: Comparison of coefficients of critical buckling load 'n' obtained from the developed program with those of Ibearugbulem et al. (2014)

Aspect Ratio; S = b/a	CCSS Plate			CCCS Plate		
	Program $N_x = n_{2x}\frac{D}{b^2}$; n	Ibearugbulem et al.(2014) n_1	% difference $100(n - n_1)/n_1$	Program $N_x = n_{2x}\frac{D}{b^2}$; n_2	Ibearugbulem et al.(2014) n_3	% difference $100(n_2 - n_3)/n_3$
1.0	6.559	6.559	0.00	9.051	9.053	-0.022
1.2	6.845	6.845	0.00	8.616	8.617	-0.012
1.5	8.037	8.037	0.00	9.216	9.218	-0.022
1.6	8.582	8.582	0.00	9.633	9.635	-0.021
2.0	11.347	11.347	0.00	12.066	12.068	-0.017
Aver. %diff.			0.00			-0.019

From Table 4.77, the results of critical buckling load for CCSS and CCCS plate are compared with those of existing research works. The values critical buckling load for CCSS plate were compared with those obtained by Ibearugbulem et al. (2014), and the percentage differences are 0.000%, 0.000%, 0.000%, 0.000% and 0.000% respectively for aspect ratio 1.0, 1.2, 1.5, 1.6, and 2.0. The average percentage differences is 0.000%. These results indicate no difference, and thus agree very absolutely.

Also for CCCS plate, values of critical buckling load obtained from the program were compared with the same authors in Table 4.77, and the percentage differences are -0.022%, -0.012%, -0.022%, -0.021% and -0.017%, with an average percentage of -0.019%. These are also very close and show only insignificant difference. Thus, this implies that this study program, is accurate and adequate for analyzing CCSS and CCCS rectangular plates.

Table 4.78: Comparison of coefficients of critical buckling load 'n' obtained from the developed program with those of Ibearugbulem et al. (2014).

Aspect Ratio; S = b/a	SSFS Plate			SCFS Plate		
	Program $N_x = n_{2x}\frac{D}{b^2}$; n	Ibearugbulem et al.(2014) n_1	% difference $100(n - n_1)/n_1$	Program $N_x = n_{2x}\frac{D}{b^2}$; n_2	Ibearugbulem et al.(2014) n_3	% difference $100(n_2 - n_3)/n_3$
1.0	1.562	1.347	15.961	2.682	2.183	22.858
1.2	1.988	1.773	12.126	3.606	3.107	16.061
1.5	2.788	2.573	8.356	5.319	4.820	10.353
1.6	3.095	2.881	7.428	5.977	5.478	9.109
2.0	4.531	4.316	4.981	9.035	8.536	5.846
Aver. %diff.			9.771			12.845

In Table 4.78, the results of critical buckling load were compared with available literatures to justify them. Those obtained for SSFS plate were compared with those obtained by Ibearugbulem et al. (2014), and the percentage differences for are 15.961%, 12.126%, 8.356%, 7.428% and 4.981% respectively aspect ratio 1.0, 1.2, 1.5, 1.6, and 2.0. And an average percentage difference is 9.771%. These values show an upper bound to those under comparison, and are little above 5% difference but within the acceptable limit in statistics.

Also, from the same Table 4.78, the values of critical buckling load for SCFS plate, were compared with the same authors and the percentage differences are 22.858%, 16.061%, 10.353%, 9.109% and 5.846%, with an average value of 12.845%. These values are upper bound to those compared with, and are above 5%, but still falls within statistically acceptable limit. The difference is because of some modifications introduced in the derivation of shape function with free edge, which modifies the shape function, and involve more rigourous manual computations, that may have increase the error factor.

Thus, this implies that, this study program, is accurate and useful for analyzing SSFS and SCFS rectangular plates.

Table 4.79: Comparison of coefficients of critical buckling load 'n' obtained from the developed program with those of Ibearugbulem et al. (2014)

Aspect Ratio S = b/a	CSFS Plate			CCFS Plate		
	Program $N_x = n_{2x}\frac{D}{b^2}$; n	Ibearugbulem et al.(2014) n_1	% difference $100(n-n_1)/n_1$	Program $N_x = n_{2x}\frac{D}{b^2}$; n_2	Ibearugbulem et al.(2014) n_3	% difference $100(n_2-n_3)/n_3$
1.0	1.957	1.717	13.978	3.029	2.792	8.489
1.2	2.275	2.038	11.629	3.861	3.624	6.540
1.5	2.987	2.751	8.579	5.499	5.262	4.504
1.6	3.277	3.040	7.796	6.140	5.903	4.015
2.0	4.664	4.427	5.354	9.156	8.920	2.646
Aver. %diff.			9.467			5.239

The coefficients of critical buckling load in Table 4.79, for CSFS plate, were compared with those obtained by Ibearugbulem et al. (2014), and the percentage differences are 13.978%, 11.629%, 8.579%, 7.796% and 5.354% respectively for aspect ratio 1.0, 1.2, 1.5, 1.6, and 2.0. And and average percentage difference is 9.467%. These values indicate an upper bound to those under comparison, and are little above 5% difference but are within the acceptable limit in statistics.

Also, those coefficients for CCFS plate in the same Table 4.79, were compared with the same authors and the percentage differences are 8.489%, 6.540%, 4.504%, 4.015% and 2.646%, with an average value of 5.239%. These are upper bound to those compared with, and are slightly above 5%, but there are statistically acceptable.

Thus, this means that this study program, is okay and quicker way for analyzing CSFS and CCFS rectangular plates.

Table 4.80: Comparison of coefficients of critical buckling load 'n' obtained from the developed program with those of Ibearugbulem et al. (2014)

Aspect Ratio S = b/a	SCFCPlate			CCFC Plate		
	Program $N_x = n_{2x}\frac{D}{b^2}$; n	Ibearugbulem et al.(2014) n_1	% difference $100(n - n_1)/n_1$	Program $N_x = n_{2x}\frac{D}{b^2}$; n_2	Ibearugbulem et al.(2014) n_3	% difference $100(n_2 - n_3)/n_3$
1.0	4.808	4.593	4.681	5.139	4.902	4.835
1.2	6.668	6.454	3.316	6.912	6.675	3.551
1.5	10.106	9.891	2.174	10.279	10.042	2.360
1.6	11.423	11.208	1.918	11.580	11.343	2.089
2.0	17.545	17.331	1.235	17.663	17.426	1.360
Aver. %diff.			2.665			2.839

In Table 4.80, the coefficients of critical buckling load, n, for SCFC plate, were compared with those gotten from Ibearugbulem et al. (2014), and the percentage differences are 4.681%, 3.316%, 2.174%, 1.918% and 1.235% respectively for aspect ratio 1.0, 1.2, 1.5, 1.6, and 2.0. The average percentage difference is 2.665%. These values show that there are upper bound to those under comparison, and are below 5%. Thus, there are very close to each other.

Also, those for CCFC plate, in the same Table4.80, were compared with the same authors. And the percentage differences are 4.835%, 3.551%, 2.360%, 2.089% and 1.360% with an average value of 2.839%. All the values are upper bound to those compared with, and are below 5%. This shows that the values agree closely with each other. Thus, this means that this study program, is accurate and adequate for analyzing SCFC and CCFC rectangular plates.

4.2.3 Discussions of the Results of Free Vibration Analysis

In order to validate these results, obtained from this new program, the coefficients of fundamental natural frequency, were compared in Tables 4.81–4.86. This comparison was made between these results with those available in literatures.

Table4.81: Comparison of coefficients of fundamental frequency 'f' obtained from the developed program with those of Ibearugbulem et al. (2014)

Aspect Ratio S = b/a	SSSS Plate			CCCC Plate		
	Program $\omega = \frac{f_s}{a^2}\sqrt{\frac{D}{\rho h}}$; $f = f_s/\pi^2$	Ibearugbulem et al.(2014) f_1	% difference $100(f- f_1)/ f_1$	Program $\omega = \frac{f_s}{a^2}\sqrt{\frac{D}{\rho h}}$; $f_2 = f_s/\pi^2$	Ibearugbulem et al.(2014) f_3	% difference $100(f_2- f_3)/ f_3$
1.0	2.001	2.001	0.00	3.648	3.644	0.11
1.2	1.695	1.695	0.00	3.118	3.115	0.10
1.5	1.445	1.445	0.00	2.740	2.738	0.073
1.6	1.391	1.391	0.00	2.668	2.666	0.075
2.0	1.251	1.251	0.00	2.497	2.496	0.04
Aver. %diff.			0.00			0.080

Where f_s is the coefficient of the fundamental natural frequency.

To authenticate these results, comparison was made in Table 4.81 for SSSS and CCCC plates between the coefficients of fundamental natural frequency obtained from the computer program with those obtain by Ibearugbulem et al. (2014).

For SSSS plate, the percentage differences are all 0.000%, for aspect ratio 1.0, 1.2, 1.5, 1.6, and 2.0. These values indicate no difference with those under comparison and are the same.

Also, for CCCC plate, values in the same Table 4.81, were compared with the same authors. The percentage differences are 0.110%, 0.096%, 0.073%, 0.075% and 0.040%, with an average percentage difference of 0.079%. These are slightly upper bound to those compared with, and are below 5%. This shows negligible difference with each other. Furthermore , comparing the same values with those obtained by Njoku et al. (2013) for the same aspect ratios, the percentage difference are 0.081%, 0.075%, 0.061%, 0.059% and 0.042%, with an average percentage difference of 0.064%. All are below 5%, and shows negligible difference with each other.

Following the same procedure of validation, the remaining plates are analyze as follows:

Table4.82: Comparison of coefficients of fundamental frequency 'f' obtained from the developed program with those of Ibearugbulem et al. (2014)

Aspect Ratio S = b/a	CSSS Plate			CSCS Plate		
	Program $\omega = \frac{f_s}{a^2}\sqrt{\frac{D}{\rho h}}$; $f = f_s/\pi^2$	Ibearugbulem et al.(2014) f_1	% difference $100(f - f_1)/f_1$	Program $\omega = \frac{f_s}{a^2}\sqrt{\frac{D}{\rho h}}$; $f_2 = f_s/\pi^2$	Ibearugbulem et al.(2014) f_3	% difference $100(f_2 - f_3)/f_3$
1.0	2.399	2.399	0.00	2.934	2.934	0.00
1.2	1.945	1.945	0.00	2.277	2.278	-0.044
1.5	1.584	1.584	0.00	1.762	1.762	0.00
1.6	1.508	1.508	0.00	1.656	1.656	0.00
2.0	1.316	1.316	0.00	1.390	1.391	-0.072
Aver. %diff.			0.00			-0.023

Table 4.82, contains the comparison between the values of fundamental natural frequency obtained for CSSS plate from the program and those of Ibearugbulem et al. (2014). The percentage differences are 0.000%, 0.000%, 0.000%, 0.000% and 0.000% respectively, for aspect ratio 1.0, 1.2, 1.5, 1.6, and 2.0. These values indicate no difference with those under comparison and are adequate.

Also, those for CSCS plate were compared with those obtained by Ibearugbulem et al. (2014) and the percentage differences for aspect ratio 1.0, 1.2, 1.5, 1.6, and 2.0 are 0.000%, -0.044%, 0.000%, 0.000% and -0.072% respectively with an average percentage differences of 0.023%. These values show no difference with those under comparison and are the same.

Thus, this implies that this study program, is accurate and adequate for analyzing CSSS and CSCS rectangular plates.

Recalled that, fundamental natural frequency is the value of externally induced vibrating frequency on the plate that causes it to resonate. Physical resonance makes a vibrating continuum to deflect excessively and is therefore a very dangerous phenomenon. Hence, the force or external frequency should not get close to the fundamental natural frequency of a vibrating continuum during its entire life time.

Table4.83: Comparison of coefficients of fundamental frequency 'f' obtained from the developed program with those of Ibearugbulem et al. (2014)

Aspect Ratio S = b/a	CCSS Plate			CCCS Plate		
	Program $\omega = \frac{f_s}{a^2}\sqrt{\frac{D}{\rho h}}$ $f = f_s/\pi^2$	Ibearugbulem et al.(2014) f_1	% difference $100(f - f_1)/f_1$	Program $\omega = \frac{f_s}{a^2}\sqrt{\frac{D}{\rho h}}$ $f_2 = f_s/\pi^2$	Ibearugbulem et al.(2014) f_3	% difference $100(f_2 - f_3)/f_3$
1.0	2.749	2.749	0.00	3.229	3.229	0.00
1.2	2.340	2.340	0.00	2.625	2.625	0.00
1.5	2.028	2.028	0.00	2.172	2.172	0.00
1.6	1.965	1.965	0.00	2.082	2.082	0.00
2.0	1.808	1.808	0.00	1.864	1.864	0.00
Aver. %diff.			0.00			0.00

The coefficients of fundamental natural frequency in Table 4.83, for CCSS plate were compared with those obtained by Ibearugbulem et al. (2014), and the percentage differences are 0.000%, 0.000%, 0.000%, 0.000% and 0.000% with an average value of 0.000% which shows no difference with each other.

Also, those for CCCS plate in the same Table above, were compared with same authors, and the percentage differences are 0.000%, 0.000%, 0.000%, 0.000% and 0.000% respectively for aspect ratio 1.0, 1.2, 1.5, 1.6, and 2.0. And the average percentage differences of 0.000%. These values indicate no difference with those under compared with. This is indication that, the developed programs are adequate and satisfactory programs for stability analysis of the plates.

Table 4.84: Comparison of coefficients of fundamental frequency 'f' obtained from the developed program with those of Ibearugbulem et al. (2014)

Aspect Ratio S = b/a	SSFS Plate			SCFS Plate		
	Program $\omega = \frac{f_s}{a^2}\sqrt{\frac{D}{\rho h}}$; $f = f_s/\pi^2$	Ibearugbulem et al.(2014) f_1	% difference $100(f- f_1)/ f_1$	Program $\omega = \frac{f_s}{a^2}\sqrt{\frac{D}{\rho h}}$; $f_2 = f_s/\pi^2$	Ibearugbulem et al.(2014) f_3	% difference $100(f_2- f_3)/ f_3$
1.0	1.250	1.161	7.666	1.758	1.586	10.845
1.2	1.175	1.110	5.856	1.698	1.576	7.741
1.5	1.113	1.069	4.116	1.650	1.571	5.029
1.6	1.100	1.061	3.676	1.640	1.570	4.459
2.0	1.064	1.039	2.406	1.613	1.568	2.870
Aver. %diff.			4.744			6.189

Also, the coefficients of fundamental natural frequency, f, in Table 4.84 for SSFS plate were compared with those obtained by Ibearugbulem et al. (2014), and the percentage differences are 7.666%, 5.856%, 4.116%, 3.676% and 2.406% respectively for aspect ratio 1.0, 1.2, 1.5, 1.6, and 2.0, with an average percentage difference of 4.744%. These values are upper bound to those compared with. There are close to those under comparison, and are satisfactory.

Moreso, for SCFS plate, the values in same Table 8.84, were compared with the same authors, and the percentage differences are 10.845%, 7.741%, 5.029%, 4.459% and 2.870%, with an average value of 6.189%. This shows that the average percentage difference is slightly above 5% but still within statistically acceptable range.

Thus, this implies that this study program is adequate and a quicker way of analyzing SSFS and SCFS rectangular plates.

Table 4.85: Comparison of coefficients of fundamental frequency ‘f’ obtained from the developed program with those of Ibearugbulem et al. (2014)

Aspect Ratio S = b/a	CSFS Plate			CCFS Plate		
	Program $\omega = \frac{f_s}{a^2}\sqrt{\frac{D}{\rho h}}$; $f = f_s/\pi^2$	Ibearugbulem et al.(2014) f_1	% difference $100(f - f_1)/f_1$	Program $\omega = \frac{f_s}{a^2}\sqrt{\frac{D}{\rho h}}$; $f_2 = f_s/\pi^2$	Ibearugbulem et al.(2014) f_3	% difference $100(f_2 - f_3)/f_3$
1.0	1.398	1.310	6.718	1.868	1.793	4.183
1.2	1.257	1.190	5.630	1.757	1.703	3.171
1.5	1.152	1.106	4.159	1.678	1.641	2.255
1.6	1.131	1.090	3.761	1.662	1.630	1.963
2.0	1.080	1.052	2.662	1.624	1.603	1.310
Aver. %diff.			4.586			2.576

In Table 4.85, the coefficients of fundamental natural frequency, f, for CSFS plate were compared with Ibearugbulem et al. (2014), and the percentage differences are 6.718%, 5.630%, 4.159%, 3.761% and 2.662% respectively for aspect ratio 1.0, 1.2, 1.5, 1.6, and 2.0. The average percentage differences is 4.586%. The average percentage difference is below 5% and hence satisfactory.

Also, for CCFS plate, the coefficients were compared with those obtained by Ibearugbulem et al. (2014), and the percentage differences are 4.183%, 3.171%, 2.255%, 1.963% and 1.310% respectively for aspect ratio 1.0, 1.2, 1.5, 1.6, and 2.0. And the average percentage differences is 2.576%. These values are all less than 5%, which means, there are close to those under comparison.

Therefore, this indicates that this study program, is accurate and satisfactory for analyzing CSFS and CCFS rectangular plates.

Table 4.86: Comparison of coefficients of fundamental frequency 'f' obtained from the developed program with those of Ibearugbulem et al. (2014)

Aspect Ratio S = b/a	SCFC Plate			CCFC Plate		
	Program $\omega = \frac{f_s}{a^2}\sqrt{\frac{D}{\rho h}}$ $f = f_s/\pi^2$	Ibearugbulem et al.(2014) f_1	% difference $100(f - f_1)/f_1$	Program $\omega = \frac{f_s}{a^2}\sqrt{\frac{D}{\rho h}}$ $f_2 = f_s/\pi^2$	Ibearugbulem et al.(2014) f_3	% difference $100(f_2 - f_3)/f_3$
1.0	2.418	2.363	2.328	2.500	2.441	2.417
1.2	2.373	2.334	1.671	2.416	2.374	1.769
1.5	2.337	2.312	1.081	2.357	2.329	1.202
1.6	2.329	2.307	0.954	2.345	2.321	1.034
2.0	2.309	2.295	0.610	2.317	2.302	0.652
Aver. %diff.			1.329			1.415

Considering SCFC plate, coefficients of fundamental natural frequency, f, in Table 4.86, were compared with the same authors and the percentage differences are 2.328%, 1.671%, 1.081%, 0.954% and 0.610%, with an average value of 1.329%. This indicates that the average percentage difference is below 5% and satisfactory.

Also, for CCFC plate the coefficients in the same Table 8.86, were compared with same authors, and the percentage differences are 2.417%, 1.769%, 1.202%, 1.034% and 0.652% respectively for aspect ratio 1.0, 1.2, 1.5, 1.6, and 2.0 with an average percentage differences of 1.415%. All the percentage differences are below 5% and hence adequate.

Thus, this implies that this study Program is accurate and adequate alternative for analyzing SCFC and CCFC rectangular plates.

4.3 Discussions of Results of the Analysis of Continuous Plate

4.3.1 Discussions of Results of the Analysis of One-Way Continuous plate

To validate the results obtained from these present program, the coefficients of fixed edge moments and final support moment, from the developed program, were compared with those of manual method as presented on Table 4.87 for continuous plate in one-way.

Table 4.87: Comparison of FEM and Support Moment (SPTM) obtained from manual method with those obtained from Program for 1-way. s= 1, 4 spans

FEM = $\beta_1 qa^2$ β_1	FEM = $\beta_2 qa^2$ β_2	Percentage Difference $100(\beta_2 - \beta_1)/\beta_1$	SPTM= $\beta_3 qa^2$ β_3	SPTM = $\beta_4 qa^2$ β_4	Percentage Difference $100(\beta_4 - \beta_3)/\beta_3$
0.0000	0.0000	0.00	0.00000	0.00000	0.00
-0.06760	-0.06760	0.00	-0.06590	-0.06590	0.00
-0.06363	-0.06363	0.00	-0.06136	-0.06135	0.016
-0.06363	-0.06363	0.00	-0.06250	-0.06249	0.016
-0.06363	-0.06363	0.00	-0.06476	-0.06476	0.00
-0.06363	-0.06363	0.00	-0.06590	-0.06590	0.00
-0.06760	-0.06760	0.00	-0.06930	-0.06930	0.00
0.0000	-0.00000	0.00	0.00000	0.00000	0.00

Where,

β_1 and β_3 are the cofficients of fixed end moment and support moment respectively obtained from manual approach, and β_2 and β_4 are the cofficients of fixed end moment and support moment respectively obtained from the developed Program.

From continuous plate in one-way, the comparison in Table 4.87 indicate that there is no difference between the values obtained from both the program and those of the manual approach for FEM. The two approaches agree absolutely with each other. While a maximum percentage difference of 0.016% is indicated for final support moments. This means that the results from the program are very close to those obtained using manual approach.

Also, looking at the figures obtained for both the fixed edge and final support moments, it is very clear that the values of the final support moments indicate a good distribution of the fixed edge moments at the supports. The implication of this is that, polynomial shape functions are better approximation of the deflection shape of the individual panels, that constitute the continuous plate in one-way. It is also obvious that, the use of polynomial shape functions in the analysis of one–way continuous plate manually, is much easier, straight forward and quicker than the use of trigonometric shape functions or approach which is very tedious, and full of ambiquous manipulations, that usually lead to errors and inconsistent results.

Furthermore, with the developed program which has proved accurate and reliable, has simplify the entire cumbersome process of analysis of one-way continuous plate. This program makes it possible for the user to choose practical dimensions of the individual panel, that make up the continuous plate unlike the use of manual approach, which is mostly confined to square panels and beyond which, it becomes near impossible to carry out the tedious manipulations. It is therefore, an opportunity for analysts to utilize this program to enhance their work and safe time and effort.

In addition, from the results of continuous plates obtained, expression have been formulated for the fixed end moments, support moments and span moments for one-way continuous plates as shown in tables 4.88.

Table 4.88: Expression for FEMs, Support Moments and Span Moments of a four span one –way continuous plate, s = b/a =1.

Support	FEM =-βqa^2 -β	SPTM =-βqa^2 -β	Span	SPNM =-βqa^2 B	Expresions
1			1	0.0932	FEM = -$qa^2/10$ SPTM = -$qa^2/10$ SPNM = $qa^2/10$
2	0.0656	0.0636	2	0.0614	
3	0.0636	0.0636	3	0.0594	
4	0.0656	0.0676	4	0.0912	

	-0.1= -1/10	-0.1= -1/10		0.1= 1/10	

Table 4.88 above shows that the fixed edge moment of a one-way continuous plate, can be taken as $-qa^2/10$, in each of the clamped edge of the panels. In beam analysis, this is - or $+ql^2/12$ and - or $+ql^2/8$, for a beam fixed at both edges and for a beam fixed at one end and pin at the other end respectively. It is also clear that, both the support moments and span moments can be calculated using $-qa^2/10$. This will simplified the entire calculation process manually.

4.3.2 Discussions of Results of the Analysis of Two-Way Continuous plate

In order to authenticate the suitablility of the polynomial shape function in analysis of two-way continuous plates and also validating the results of the program, the results of both the manual approach and those of the program were compared as follows.

Table 4.89: Comparison of FEM and Support Moment (SPTM) obtained from manual method with those obtained from Program for four span two-way continuous plate. s= 1, Section S-S.

FEM = $\beta_1 qa^2$ β_1	FEM = $\beta_2 qa^2$ β_2	Percentage Difference $100(\beta_2- \beta_1)/ \beta_1$	SPTM = $\beta_3 qa^2$ β_3	SPTM = $\beta_4 qa^2$ β_4	Percentage Difference $100(\beta_4- \beta_3)/ \beta_3$
0.0000	0.0000	0.00	0.00000	0.0000	0.00
-0.05042	-0.05042	0.00	-0.05085	-0.05085	0.00
-0.05142	-0.05142	0.00	-0.05199	-0.05199	0.00
-0.05142	-0.05142	0.00	-0.05171	-0.05171	0.00
-0.05142	-0.05142	0.00	-0.05113	-0.05114	0.020
-0.05142	-0.05142	0.00	-0.05085	-0.05085	0.00
-0.05042	-0.05042	0.00	-0.04999	0.04999	0.00
0.0000	0.0000	0.00	0.00000	0.0000	0.00

where, β_1 and β_3 are the cofficients of fixed end moment and support moment respectively obtained from manual approach, and β_2 and β_4 are the cofficients of fixed end moment and support moment respectively obtained from the developed Program.

From a four span two-way continuous plate, the results presented in Table 4.89 for section S-S in x-direction, show that there is no difference between the values obtained from the computer program and those obtained by manual approach for fixed edge moments (FEM). This justifies the fact that the computer program is accurate and adequate for use in analysis of continuous plate. A maximum percentage difference of 0.020% for final support moment, is observed which is considered insignificant. This means that the values from the program are very close to those of manual approach. It also indicates that the final support moments distribute effectively the moments at the supports.

Table 4.90: Comparison of FEM and Support Moment (SPTM) obtained from manual method with those obtained from Program for a four span two-way continuous plate. S= 1, Section T-T.

FEM = $\beta_1 qa^2$ β_1	FEM = $\beta_2 qa^2$ β_2	Percentage Difference $100(\beta_2 - \beta_1)/\beta_1$	SPTM = $\beta_3 qa^2$ β_3	SPTM = $\beta_4 qa^2$ β_4	Percentage Difference $100(\beta_4 - \beta_3)/\beta_3$
0.0000	0.0000	0.00	0.00000	0.0000	0.00
-0.03856	-0.03857	0.00	-0.04026	-0.04027	0.025
-0.04252	-0.04253	0.00	-0.04478	-0.04480	0.045
-0.04252	-0.04253	0.00	-0.04365	-0.04367	0.046
-0.04252	-0.04253	0.00	-0.04139	-0.04140	0.024
-0.04252	-0.04253	0.00	-0.04024	-0.04027	0.074
-0.03856	-0.0387	0.00	-0.03686	-0.03686	0.00
0.0000	0.0000	0.00	0.00000	0.0000	0.00

Also, the comparison in Table 4.90 for section T-T in x-direction indicate that there is also no difference between the values obtained from the computer program and those obtained by manual approach for fixed edge moments (FEM). This justifies the fact that the computer program is accurate and adequate for use in analysis of continuous plate. A maximum percentage difference of 0.074% for final support moment is observed which is considered insignificant. This means that, the values from the program are very close to those of manual approach. Also, it indicates that, the final support moments distribute effectively the moments at the supports.

Table 4.91: Comparison of FEM and Support Moment (SPTM) obtained from manual method with those obtained from Program for a three span two-way continuous plate. s= 1, Section 1-1.

FEM = $\beta_1 qa^2$ β_1	FEM= $\beta_2 qa^2$ β_2	Percentage Difference $100(\beta_2 - \beta_1)/\beta_1$	SPTM = $\beta_3 qa^2$ β_3	SPTM = $\beta_4 qa^2$ β_4	Percentage Difference $100(\beta_4 - \beta_3)/\beta_3$
0.0000	0.00000	0.00	0.00000	0.00000	0.00
-0.05042	-0.05042	0.00	-0.05102	-0.05102	0.00
-0.05142	-0.05142	0.00	-0.05182	-0.05182	0.00
-0.05142	-0.05142	0.00	-0.05102	-0.05102	0.00
-0.05042	-0.05042	0.00	-0.04982	0.04982	0.00
0.00000	0.00000	0.00	0.00000	0.00000	0.00

Considering the results obtained for the strips in y-direction as shown in Table 4.91. The percentage differences between the results obtained from the computer program and those obtained by manual approach are all zeros for both fixed end moments and final support moments. This is an indication that there are little or no differences between them.

Table 4.92: Comparison of FEM and Support Moment (SPTM) obtained from manual method with those obtained from Program for a three span two-way continuous plate. S = 1, Section 2-2

FEM = $\beta_1 qa^2$ β_1	FEM = $\beta_2 qa^2$ β_2	Percentage Difference $100(\beta_2 - \beta_1)/\beta_1$	SPTM = $\beta_3 qa^2$ β_3	SPTM = $\beta_4 qa^2$ β_4	Percentage Difference $100(\beta_4 - \beta_3)/\beta_3$
0.0000	0.00000	0.00	0.00000	0.00000	0.00
-0.03856	-0.03857	0.026	-0.04094	-0.04095	0.024
-0.04252	-0.04253	0.024	-0.04410	-0.04412	0.045
-0.04252	-0.04253	0.024	-0.04094	-0.04095	0.024
-0.03856	-0.03857	0.026	-0.03618	-0.03618	0.00
0.00000	0.00000	0.00	0.00000	0.00000	0.00

In Table 4.92, comparison was made between the cofficients of FEMs and span moment, it indicates maximum percentage differences of 0.026% and 0.045% for fixed end and final support moments respectively. These are considered insignificant.

The results indicate that, polynomial shape functions can be use in analysis of two-way continuous plate. The offer less difficulties and are less cumbersome compare to the the use of trigonometric or Fourier series, which are full of assumptions that often lead to errors in results. Also, the use of polynomial shape functions, offers a better understanding in the entire analytic process unlike the former which is based more on assumption and trial and error approach.

Moreover, from the available literatures, the use of the former rigorous approach, limits continuous plate analysis to aspect ratio of one only, that is, assummimg all the individual panels, that make up the continuous plate, to only square plates. Because, the situation could be complex, when the panels of the continuous plate are not square panels. This could have be the reason why there are limited literatures in continuous plate analysis, since the former approach could not give room for practical situations in terms of dimensions. However, with the computer program developed in this study, such bottle-necks are illuminated, because this computer program offers high degree of flexibility in terms of dimensions. It can accommodate any dimension at all. It is quicker and straight forward.

From the results of continuous plates obtained, equations or expressions have been formulated for the fixed end moments, support moments and span moments, for continuous plates in two ways as shown in Table 4.93.

Table 4.93: Expressions for FEM, Support Moments and Span Moments of a 4 x 3 spans two-way Continuous Plate. s = b/a =1.

Support/Span	FEM_E=-βqa^2 -β	FEM_I=-βqa^2 -β	$SPTM_E$ =-βqa^2 -β	$SPTM_I$ =-βqa^2 -β	$SPNM_E$=-βqa^2 β	$SPNM_I$=-βqa^2 β
	Sect. S-S	Sect. T-T	Sect. S-S	Sect. T-T	Sect. S-S	Sect. T-T
1					0.09929	0.1037
2	0.05042	0.03856	0.05142	0.04252	0.07358	0.08248
	0.05142	0.04252				
3	0.05142	0.04252	0.05142	0.04252	0.07408	0.08447
	0.05142	0.04252				
4	0.05142	0.04252	0.05042	0.03855	0.09979	0.10573
	0.05042	0.03856				
	FEM= 0.1 =1/10	$FEM_{2,1}$= 0.04=1/25 $FEM_{2,3}$=0.43= 1/23	$SPTM_E$ =0.1=1/10	$SPTM_I$ =0.43 = 1/23	$SPNM_E$ =0.1 = 1/10	$SPNM_I$ = 0.1 =1/10
Expressions	FEM_E = -qa^2/10, FEM_I = -qa^2/25,-qa^2/23 Respectively. $SPTM_E$ = -qa^2/10, $SPTM_I$ = -qa^2/23 $SPNM_E$ = qa^2/10, $SPNM_I$ = qa^2/10, (E is External strip, I is Internal Strip)					

For a two-way continuous plate, in Table 4.93, it can be deduced that, the fixed end moment, of the external strips in x-direction is -qa^2/10, while that of the internal strips is -qa^2/25., for the panels pinned at one end and clamped at the other end, and -qa^2/23, for the panels fixed at both ends. The support moment for the external strips is -qa^2/10, and for internal strips is -qa^2/23. While the span moments, can be calculated from qa^2/10, in both the external and internal strips. These expressions will help simplify the manual calculations of continuous plate.

CHAPTER FIVE

CONCLUSION AND RECOMMENDATIONS

5.1 Conclusion

In this study, rectangular isotropic single panel plates and continuous plates were analyzed using polynomial series formulated shape function in Ritz energy equation, and computer programs were developed for these analyses.

From this work, the following conclusions are made:

(i) Computer programs were developed for the analysis of single panel rectangular plates in pure bending with various boundary conditions.

(ii) Also, programs were developed for stability analysis of single panel rectangular plates in pure bending with various boundary conditions.

(iii) Computer programs were developed for free vibration analysis of single panel rectangular plates with various boundary conditions.

(iv) Expression were derived based on polynomial shape function to analyze continuous plate.

(v) Programs were developed for the analysis of both one-way and two-way continuous plates.

And from the results and discussions in chapter four, it is evident that the percentage differences between the results obtained from this new programs, based on polynomial shape functions, using MATLAB programming language, for single panel thin isotropic rectangular plates, and those obtained by earlier studies are mostly insignificant, and within the acceptance limit in statistics. Therefore, considered adequate and provide a better alternative and quicker approach to analyze thin isotropic rectangular plates.

Furthermore, from the results and discussions on continuous plates, programs developed offer a better, simpler and faster way of analyzing both one-way and two-way continuous plate. The exact (manual) and approximate analysis of continuous plate was based only on aspect ratio of one only due to difficulties in handling the complex differential equations and making erroneous assumptions, but this present programs offers flexibility to work, based on dimensions of the plate which may not necessarily be square. In addition, the derived expressions from this study, can simplify even the manual computation of continuous plates.

Also, it is concluded that, polynomial shape functions are better approximations of the individual deflected shape, and are adequate and simpler for continuous plate analysis.

Therefore, the conclusion that, these developed programs, should be use for analysis of single panel and continuous rectangular plates.

5.2 Recommendations

Serious and committed effort has been made in this study, to develop computer programs for analysis of thin isotropic rectangular plates both single panel and continuous plates. Hence, the following recommendations are made:

(i) These programs are recommended for analysis of thin isotropic rectangular plates as a better and faster alternative.

(ii) The programs also should be use for analysis of continuous plate in both one-way and two-way continuous plate.

(iii) Polynomial shape functions are adequate, simpler and quicker means of analyzing continuous rectangular plates.

(iv) Future studies should consider developing programs for force vibration.

(v) Effort should be made in future studies to develop programs for orthotropic rectangular plates and others.

(vi) Also, future studies should consider expanding and improving on these programs to convert them into an installable computer software for rectangular plate analysis.

5.3 Contributions to Knowlegde

The present study has contributed to knowledge in the following ways.

(i) This work has developed computer programs for pure bending, buckling and free vibration analyses of twelve single panel rectangular isotropic plate types with various boundary conditions, based on polynomial shape functions. The program offer better, simpler, faster and more accurate way of analyzing rectangular isotropic plates.

(ii) It has derived expressions based on polynomial shape functions and developed computer programs for analysis of both one-way and two-way continuous plates.

(iii) This work has shown that, polynomial shape functions are adequate for analysis of continuous rectangular plates, and that the use of polynomial shape functions offers a simpler, quicker and accurate approach of analysis of continuous rectangular plates.

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APPENDIX 1
SINGLE PANEL RECTANGULAR PLATES PROGRAMS

3. 3.1(i) SSSS Plate

```
clc
%PROGRAM FOR SSSS PLATE
syms r q
U = r-2*r^3+r^4;
V = q-2*q^3+q^4;
diff(U,2);
(diff(U,2))^2;
y1 = int((diff(U,2))^2,r,0,1);
z1 = int(V^2,q,0,1);
Y1 = y1*z1;
diff(V,2);
(diff(V,2))^2;
y2 = int(U^2,r,0,1);
z2 = int((diff(V,2))^2,q,0,1);
Y2 = y2*z2;
diff(U,1);
diff(V,1);
y3 = int((diff(U,1))^2,r,0,1);
z3 = int((diff(V,1))^2,q,0,1);
Y3 = y3*z3;
y4 = int(U,r,0,1);
```

```
z4 = int(V,q,0,1);
Y4 = y4*z4;
z5 = int(V^2,q,0,1);
Y5 = y3*z5;
Y6 = y2*z1;
v = input('Enter value of poission ratio v:');
a = input('Enter the horizontal dimension a(m):');
b = input('Enter the vertical dimension b(m):');
Q = input('Enter the udl Q(kN/m):');
h = input('Enter the thickness h(m):');
E = input('Enter the value of young modulus E:');
p = input('Enter the value of specific density p:');
echo on
s = b/a
echo off
u = vpa(Y4/(Y1+(2*Y3/s^2)+(Y2/s^4)),5)
%The flexural Rigidity of plate D is
D = E*h^3/(12*(1-v^2));
r = input('Enter value of r for deflection r:');
q = input('Enter value of q for deflection q:');
k = (r-2*r^3+r^4)*(q-2*q^3+q^4);
%Amplitude is
A = u*Q*a^4/D;
%deflection max
alpha = vpa((u*k),7)
Wmax = vpa((alpha*Q*a^4/D),7);
%Center Moment
r1 = input('Enter value of r for center moment r1:');
q1 = input('Enter value of q for center moment q1:');
beta = vpa(-u*((12*r1^2-12*r1)*(q1-2*q1^3+q1^4)+(v/s.^2)*(r1-2*r1^3+r1^4)*(12*q1^2-
12*q1)),5)
Mxc = vpa(beta*Q*a^2,5);
beta1 = vpa(-u*(v*(12*r1^2-12*r1)*(q1-2*q1^3+q1^4)+(1/s.^2)*(r1-
2*r1^3+r1^4)*(12*q1^2-12*q1)),5)
```

```
Myc = vpa(beta1*Q*a^2,5);
%Fixed edge moment
r2 = input('Enter the value for edge moment in X-D r2:');
q2 = input('Enter the value for edge moment in X-D q2:');
r3 = input('Enter the value for edge moment in Y-D r3:');
q3 = input('Enter the value for edge moment in y-D q3:');
beta2 = vpa(-u*((12*r2^2-12*r2)*(q2-2*q2^3+q2^4)+(v/s.^2)*(r2-2*r2^3+r2^4)*(12*q2^2-
12*q2)),5)
Mxe = vpa(beta2*Q*a^2,5);
beta3 = vpa(-u*(v*(12*r3^2-12*r3)*(q3-2*q3^3+q3^4)+(1/s.^2)*(r3-
2*r3^3+r3^4)*(12*q3^2-12*q3)),5)
Mye = vpa(beta3*Q*a^2,5);
%max Shear Force
r4 = input('Enter the value for Vx r4:');
q4 = input('Enter the value for Vx q4:');
r5 = input('Enter the value for Vy r5:');
q5 = input('Enter the value for Vy q5:');
delta =vpa(-u*((24*r4-12)*(q4-2*q4^3+q4^4)+((2-v)/s.^2)*(1-6*r4^2+4*r4^3)*(12*q4^2-
12*q4)),5)
Vxmax = vpa(delta*Q*a,5);
delta1 = vpa(-u*(((2-v)/s.^1)*(12*r5^2-12*r5)*(1-6*q5^2+4*q5^3)+(1/s.^3)*(r5-
2*r5^3+r5^4)*(24*q5-12)),5)
Vymax = vpa(delta1*Q*a,5);
%slope
r6 = input('Enter the value for slopex r6:');
q6 = input('Enter the value for slopex q6:');
r7 = input('Enter the value for slopey r7:');
q7 = input('Enter the value for slopey q7:');
yr = vpa((u*(1-6*r6^2+4*r6^3)*(q6-2*q6^3+q6^4))/D,5)
slopex = vpa(yr*Q*a^3,5);
yq = vpa((u*(r7-2*r7^3+r7^4)*(1-6*q7^2+4*q7^3))/(s*D),5)
Slopey = vpa(yq*Q*a^3,5);
%Bulkling Load or Resistant of the plate
n = vpa((Y1+(2*Y3/s^2)+(Y2/s^4))/Y5,5)
```

```
%in term Nx = n1*D/b^2
n1 = n*s^2
n2 = n1/pi^2
Nx = vpa((n*D/a^2),7);
%Fundamental Natural frequency
f = vpa(sqrt((Y1+(2*Y3/s^2)+(Y2/s^4))/Y6),5)
f1 = f/pi^2
F = vpa((f/a^2)*sqrt(D/p*h),7);
```

3. 3.1(ii) CCCC Plate

```
clc
%PROGRAM FOR CCCC PLATE
syms r q
U = r^2-2*r^3+r^4;
V = q^2-2*q^3+q^4;
diff(U,2);
(diff(U,2))^2;
y1 = int((diff(U,2))^2,r,0,1);
z1 = int(V^2,q,0,1);
Y1 = y1*z1;
diff(V,2);
(diff(V,2))^2;
y2 = int(U^2,r,0,1);
z2 = int((diff(V,2))^2,q,0,1);
Y2 = y2*z2;
diff(U,1);
diff(V,1);
y3 = int((diff(U,1))^2,r,0,1);
z3 = int((diff(V,1))^2,q,0,1);
Y3 = y3*z3;
y4 = int(U,r,0,1);
z4 = int(V,q,0,1);
Y4 = y4*z4;
z5 = int(V^2,q,0,1);
```

```
Y5 = y3*z5;
Y6 = y2*z1;
v = input('Enter value of poission ratio v:');
a = input('Enter the horizontal dimension a(m):');
b = input('Enter the vertical dimension b(m):');
Q = input('Enter the udl Q(kN/m):');
h = input('Enter the thickness h(m):');
E = input('Enter the value of young modulus E:');
p = input('Enter the value of specific density p:');
echo on
s = b/a
echo off
u = vpa(Y4/(Y1+(2*Y3/s^2)+(Y2/s^4)),5)
%The flexural Rigidity of plate D is
D = E*h^3/(12*(1-v^2));
r = input('Enter value of r for deflection r:');
q = input('Enter value of q for deflection q:');
k = (r^2-2*r^3+r^4)*(q^2-2*q^3+q^4);
%Amplitude is
A = u*Q*a^4/D;
%deflection max
alpha = vpa((u*k),7)
Wmax = vpa((alpha*Q*a^4/D),7)
%Center Moment
r1 = input('Enter value of r for center moment r1:');
q1 = input('Enter value of q for center moment q1:');
beta = vpa(-u*((2-12*r1+12*r1^2)*(q1^2-2*q1^3+q1^4)+(v/s.^2)*(r1^2-2*r1^3+r1^4)*(2-
12*q1+12*q1^2)),7)
Mxc = vpa(beta*Q*a^2,5)
beta1 = vpa(-u*((v*(2-12*r1+12*r1^2)*(q1^2-2*q1^3+q1^4))+(1/s.^2)*(r1^2-
2*r1^3+r1^4)*(2-12*q1+12*q1^2)),7)
myc = vpa(beta1*Q*a^2,7)
%Fixed edge moment
r2 = input('Enter the value for edge moment in X-D r2:');
```

```
q2 = input('Enter the value for edge moment in X-D q2:');
r3 = input('Enter the value for edge moment in Y-D r3:');
q3 = input('Enter the value for edge moment in Y-D q3:');
beta2 = vpa(-u*((2-12*r2+12*r2^2)*(q2^2-2*q2^3+q2^4)+(v/s.^2)*(r2^2-2*r2^3+r2^4)*(2-
12*q2+12*q2^2)),7)
Mxe = vpa(beta2*Q*a^2,7)
beta3 = vpa(-u*((v*(2-12*r3+12*r3^2)*(q3^2-2*q3^3+q3^4))+(1/s.^2)*(r3^2-
2*r3^3+r3^4)*(2-12*q3+12*q3^2)),7)
Mye = vpa(beta3*Q*a^2,7)
%Shear Force
r4 = input('Enter the value for Vx r4:');
q4 = input('Enter the value for Vx q4:');
r5 = input('Enter the value for Vy r5:');
q5 = input('Enter the value for Vy q5:');
delta =vpa(-u*((24*r4-12)*(q4^2-2*q4^3+q4^4)+((2-v)/s^3)*(2*r4-6*r4^2+4*r4^3)*(2-
12*q4+12*q4^2)),7)
Vx = vpa(delta*Q*a,7)
delta1 = vpa(-u*((((2-v)/s.^1)*(2-12*r5+12*r5^2)*(2*q5-6*q5^2+4*q5^3)+(1/s.^3)*(r5^2-
2*r5^3+r5^4)*(24*q5-12)),7)
Vy = vpa(delta1*Q*a,7)
%slope
r6 = input('Enter the value for slopex r6:');
q6 = input('Enter the value for slopex q6:');
r7 = input('Enter the value for slopey r7:');
q7 = input('Enter the value for slopey q7:');
yr = vpa((u*(2*r6-6*r6^2+4*r6^3)*(q6^2-2*q6^3+q6^4))/D,7)
slopex = vpa(yr*Q*a^3,7)
yq = vpa((u*(r7^2-2*r7^3+r7^4)*(2*q7-6*q7^2+4*q7^3))/(s*D),7)
Slopey = vpa(yq*Q*a^3,7)
%Bulkling Load or Resistant of the plate
n = vpa((Y1+(2*Y3/s^2)+(Y2/s^4))/Y5,7)
%in term Nx = n1*D/b^2
n1 = n*s^2
n2 = n1/pi^2
```

```
Nx = vpa((n*D/a^2),7)
%Fundamental Natural frequency
f = vpa(sqrt((Y1+(2*Y3/s^2)+(Y2/s^4))/Y6),7)
f1 = f/pi^2
F = vpa((f/a^2)*sqrt(D/p*h),7)
```

3. 3.1(iii) CSSS Plate

```
clc
%PROGRAM FOR CSSS PLATE
syms r q
U = r-2*r^3+r^4;
V = 1.5*q^2-2.5*q^3+q^4;
diff(U,2);
(diff(U,2))^2;
y1 = int((diff(U,2))^2,r,0,1);
z1 = int(V^2,q,0,1);
Y1 = y1*z1;
diff(V,2);
(diff(V,2))^2;
y2 = int(U^2,r,0,1);
z2 = int((diff(V,2))^2,q,0,1);
Y2 = y2*z2;
diff(U,1);
diff(V,1);
y3 = int((diff(U,1))^2,r,0,1);
z3 = int((diff(V,1))^2,q,0,1);
Y3 = y3*z3;
y4 = int(U,r,0,1);
z4 = int(V,q,0,1);
Y4 = y4*z4;
z5 = int(V^2,q,0,1);
Y5 = y3*z5;
```

```
Y6 = y2*z1;
v = input('Enter value of poission ratio v:');
a = input('Enter the horizontal dimension a(m):');
b = input('Enter the vertical dimension b(m):');
Q = input('Enter the udl Q(kN/m):');
h = input('Enter the thickness h(m):');
E = input('Enter the value of young modulus E:');
p = input('Enter the value of specific density p:');
echo on
s = b/a
echo off
u = vpa(Y4/(Y1+(2*Y3/s^2)+(Y2/s^4)),5)
%The flexural Rigidity of plate D is
D = E*h^3/(12*(1-v^2));
r = input('Enter value of r for deflection r:');
q = input('Enter value of q for deflection q:');
k = (r-2*r^3+r^4)*(1.5*q^2-2.5*q^3+q^4);
%Amplitude is
A = u*Q*a^4/D;
%deflection max
alpha = vpa((u*k),5)
Wmax = vpa((alpha*Q*a^4/D),5)
%Center Moment
r1 = input('Enter value of r for center moment r1:');
q1 = input('Enter value of q for center moment q1:');
beta = vpa(-u*((-12*r1+12*r1^2)*(1.5*q1^2-2.5*q1^3+q1^4)+(v/s.^2)*(r1-
2*r1^3+r1^4)*(3-15*q1+12*q1^2)),5)
Mxc = vpa(beta*Q*a^2,5)
beta1 = vpa(-u*(v*(-12*r1+12*r1^2)*(1.5*q1^2-2.5*q1^3+q1^4)+(1/s.^2)*(r1-
2*r1^3+r1^4)*(3-15*q1+12*q1^2)),5)
Myc = vpa(beta1*Q*a^2,5)
%Fixed edge Moment
r2 = input('Enter the value for edge moment in X-D r2:');
q2 = input('Enter the value for edge moment in X-D q2:');
```

```
r3 = input('Enter the value for edge moment in Y-D r3:');
q3 = input('Enter the value for edge moment in Y-D q3:');
beta2 = vpa(-u*((-12*r2+12*r2^2)*(1.5*q2^2-2.5*q2^3+q2^4)+(v/s.^2)*(r2-
2*r2^3+r2^4)*(3-15*q2+12*q2^2)),5)
Mxe = vpa(beta2*Q*a^2,5)
beta3 = vpa(-u*(v*(-12*r3+12*r3^2)*(1.5*q3^2-2.5*q3^3+q3^4)+(1/s.^2)*(r3-
2*r3^3+r3^4)*(3-15*q3+12*q3^2)),5)
Mye = vpa(beta3*Q*a^2,5)
%Shear Force
r4 = input('Enter the value for Vx r4:');
q4 = input('Enter the value for Vx q4:');
r5 = input('Enter the value for Vy r5:');
q5 = input('Enter the value for Vy q5:');
delta =vpa(-u*((24*r4-12)*(1.5*q4^2-2.5*q4^3+q4^4)+((2-v)/s.^2)*(1-6*r4^2+4*r4^3)*(3-
15*q4+12*q4^2)),5)
Vx = vpa(delta*Q*a,5)
delta1 = vpa(-u*((((2-v)/s.^1)*(-12*r5+12*r5^2)*(3*q5-7.5*q5^2+4*q5^3)+((1/s.^3)*(r5-
2*r5^3+r5^4)*(24*q5-15))),5)
Vy = vpa(delta1*Q*a,5)
%slope
r6 = input('Enter the value for slopex r6:');
q6 = input('Enter the value for slopex q6:');
r7 = input('Enter the value for slopey r7:');
q7 = input('Enter the value for slopey q7:');
yr = vpa((u*(1-6*r6^2+4*r6^3)*(1.5*q6^2-2.5*q6^3+q6^4))/D,5)
slopex = vpa(yr*Q*a^3,5)
yq = vpa((u*(r7-2*r7^3+r7^4)*(3*q-7.5*q7^2+4*q7^3))/(s*D),5)
Slopey = vpa(yq*Q*a^3,5)
%Bulkling Load or Resistant of the plate
n = vpa((Y1+(2*Y3/s^2)+(Y2/s^4))/Y5,5)
%in term Nx = n1*D/b^2
n1 = n*s^2
n2 = n1/pi^2
Nx = vpa((n*D/a^2),7)
```

```
%Fundamental Natural frequency
f = vpa(sqrt((Y1+(2*Y3/s^2)+(Y2/s^4))/Y6),7)
f1 = f/pi^2
F = vpa((f/a^2)*sqrt(D/p*h),7)
```

3. 3.1(iv) CSCS Plate

```
clc
%PROGRAM FOR CSCS PLATE
syms r q
U = r-2*r^3+r^4;
V = q^2-2*q^3+q^4;
diff(U,2);
(diff(U,2))^2;
y1 = int((diff(U,2))^2,r,0,1);
z1 = int(V^2,q,0,1);
Y1 = y1*z1;
diff(V,2);
(diff(V,2))^2;
y2 = int(U^2,r,0,1);
z2 = int((diff(V,2))^2,q,0,1);
Y2 = y2*z2;
diff(U,1);
diff(V,1);
y3 = int((diff(U,1))^2,r,0,1);
z3 = int((diff(V,1))^2,q,0,1);
Y3 = y3*z3;
y4 = int(U,r,0,1);
z4 = int(V,q,0,1);
Y4 = y4*z4;
z5 = int(V^2,q,0,1);
Y5 = y3*z5;
```

```
Y6 = y2*z1;
v = input('Enter value of poission ratio v:');
a = input('Enter the horizontal dimension a(m):');
b = input('Enter the vertical dimension b(m):');
Q = input('Enter the udl Q(kN/m):');
h = input('Enter the thickness h(m):');
E = input('Enter the value of young modulus E:');
p = input('Enter the value of specific density p:');
echo on
s = b/a
echo off
u = vpa(Y4/(Y1+(2*Y3/s^2)+(Y2/s^4)),5)
%The flexural Rigidity of plate D is
D = E*h^3/(12*(1-v^2));
r = input('Enter value of r for deflection r:');
q = input('Enter value of q for deflection q:');
k = (r-2*r^3+r^4)*(q^2-2*q^3+q^4);
%Amplitude is
A = u*Q*a^4/D;
%deflection max
alpha = vpa((u*k),7)
Wmax = vpa((alpha*Q*a^4/D),7)
%Center Moment
r1 = input('Enter value of r for center moment r1:');
q1 = input('Enter value of q for center moment q1:');
beta = vpa(-u*((12*r1^2-12*r1)*(q1^2-2*q1^3+q1^4)+(v/s.^2)*(r1-2*r1^3+r1^4)*(2-
12*q1+12*q1^2)),5)
Mxc = vpa(beta*Q*a^2,5)
beta1 = vpa(-u*(v*(12*r1^2-12*r1)*(q1^2-2*q1^3+q1^4)+(1/s.^2)*(r1-2*r1^3+r1^4)*(2-
12*q1+12*q1^2)),5)
Myc = vpa(beta1*Q*a^2,5)
%Fixed edge moment
r2 = input('Enter the value for edge moment in X-D r2:');
q2 = input('Enter the value for edge moment in X-D q2:');
```

```
r3 = input('Enter the value for edge moment in Y-D r3:');
q3 = input('Enter the value for edge moment in Y-D q3:');
beta = vpa(-u*((12*r2^2-12*r2)*(q2^2-2*q2^3+q2^4)+(v/s.^2)*(r2-2*r2^3+r2^4)*(2-
12*q2+12*q2^2)),5)
Mxe = vpa(beta*Q*a^2,5)
beta1 = vpa(-u*(v*(12*r3^2-12*r3)*(q3^2-2*q3^3+q3^4)+(1/s.^2)*(r3-2*r3^3+r3^4)*(2-
12*q3+12*q3^2)),5)
Mye = vpa(beta1*Q*a^2,5)
%Shear Force
r4 = input('Enter the value for Vx r4:');
q4 = input('Enter the value for Vx q4:');
r5 = input('Enter the value for Vy r5:');
q5 = input('Enter the value for Vy q5:');
delta =vpa(-u*((24*r4-12)*(q4^2-2*q4^3+q4^4)+((2-v)/s.^2)*(1-6*r4^2+4*r4^3)*(2-
12*q4+12*q4^2)),5)
Vx = vpa(delta*Q*a,5)
delta1 = vpa(-u*((((2-v)/s.^1)*(12*r5^2-12*r5)*(2*q5-6*q5^2+4*q5^3)+((1/s.^3)*(r5-
2*r5^3+r5^4)*(24*q5-12))),5)
Vy = vpa(delta1*Q*a,5)
%slope
r6 = input('Enter the value for slopex r6:');
q6 = input('Enter the value for slopex q6:');
r7 = input('Enter the value for slopey r7:');
q7 = input('Enter the value for slopey q7:');
yr = vpa((u*(1-6*r6^2+4*r6^3)*(q6^2-2*q6^3+q6^4))/D,5)
slopex = vpa(yr*Q*a^3,5)
yq = vpa((u*(r7-2*r7^3+r7^4)*(2*q7-6*q7^2+4*q7^3))/(s*D),5)
Slopey = vpa(yq*Q*a^3,5)
%Bulkling Load or Resistant of the plate
n = vpa((Y1+(2*Y3/s^2)+(Y2/s^4))/Y5,7)
%in term Nx = n1*D/b^2
n1 = n*s^2
n2 = n1/pi^2
Nx = vpa((n*D/a^2),7)
```

```
%Fundamental Natural frequency
f = vpa(sqrt((Y1+(2*Y3/s^2)+(Y2/s^4))/Y6),7)
f1 = f/pi^2
F = vpa((f/a^2)*sqrt(D/p*h),7)
```

3. 3.1(v) CCSS Plate

```
clc
%PROGRAM FOR CCSS PLATE
syms r q
U = 1.5*r^2-2.5*r^3+r^4;
V = 1.5*q^2-2.5*q^3+q^4;
diff(U,2);
(diff(U,2))^2;
y1 = int((diff(U,2))^2,r,0,1);
z1 = int(V^2,q,0,1);
Y1 = y1*z1;
diff(V,2);
(diff(V,2))^2;
y2 = int(U^2,r,0,1);
z2 = int((diff(V,2))^2,q,0,1);
Y2 = y2*z2;
diff(U,1);
diff(V,1);
y3 = int((diff(U,1))^2,r,0,1);
z3 = int((diff(V,1))^2,q,0,1);
Y3 = y3*z3;
y4 = int(U,r,0,1);
z4 = int(V,q,0,1);
Y4 = y4*z4;
z5 = int(V^2,q,0,1);
Y5 = y3*z5;
Y6 = y2*z1;
v = input('Enter value of poission ratio v:');
a = input('Enter the horizontal dimension a(m):');
```

```
b = input('Enter the vertical dimension b(m):');
Q = input('Enter the udl Q(kN/m):');
h = input('Enter the thickness h(m):');
E = input('Enter the value of young modulus E:');
p = input('Enter the value of specific density p:');
echo on
s = b/a
echo off
u = vpa(Y4/(Y1+(2*Y3/s^2)+(Y2/s^4)),5)
%The flexural Rigidity of plate D is
D = E*h^3/(12*(1-v^2));
r = input('Enter value of r for deflection r:');
q = input('Enter value of q for deflection q:');
k = (1.5*r^2-2.5*r^3+r^4)*(1.5*q^2-2.5*q^3+q^4);
%Amplitude is
A = u*Q*a^4/D;
%deflection max
alpha = vpa((u*k),7)
Wmax = vpa((alpha*Q*a^4/D),7)
%Center Moment
r1 = input('Enter value of r for center moment r1:');
q1 = input('Enter value of q for center moment q1:');
beta = vpa(-u*((3-15*r1+12*r1^2)*(1.5*q1^2-2.5*q1^3+q1^4)+(v/s.^2)*(1.5*r1^2-
2.5*r1^3+r1^4)*(3-15*q1+12*q1^2)),7)
Mxc = vpa(beta*Q*a^2,7)
beta1 = vpa(-u*(v*(3-15*r1+12*r1^2)*(1.5*q1^2-2.5*q1^3+q1^4)+(1/s.^2)*(1.5*r1^2-
2.5*r1^3+r1^4)*(3-15*q1+12*q1^2)),7)
Myc = vpa(beta1*Q*a^2,7)
%Fixed edge moment
r2 = input('Enter the value for edge moment in X-D r2:');
q2 = input('Enter the value for edge moment in X-D q2 :');
r3 = input('Enter the value for edge moment in Y-D r3:');
q3 = input('Enter the value for edge moment in Y-D q3:');
```

```
beta2 = vpa(-u*((3-15*r2+12*r2^2)*(1.5*q2^2-2.5*q2^3+q2^4)+(v/s.^2)*(1.5*r2^2-
2.5*r2^3+r2^4)*(3-15*q2+12*q2^2)),7)
Mxe = vpa(beta2*Q*a^2,7)
beta3 = vpa(-u*(v*(3-15*r3+12*r3^2)*(1.5*q3^2-2.5*q3^3+q3^4)+(1/s.^2)*(1.5*r3^2-
2.5*r3^3+r3^4)*(3-15*q3+12*q3^2)),7)
Mye = vpa(beta3*Q*a^2,5)
%max Shear Force
r4 = input('Enter the value for Vx r1:');
q4 = input('Enter the value for Vx q1:');
r5 = input('Enter the value for Vy r2:');
q5 = input('Enter the value for Vy q2:');
delta =vpa(-u*((24*r4-15)*(1.5*q4^2-2.5*q4^3+q4^4)+((2-v)/s.^2)*(3*r4-
7.5*r4^2+4*r4^3)*(3-15*q4+12*q4^2)),7)
vxmax = vpa(delta*Q*a,7)
delta1 = vpa(-u*(((2-v)/s.^1)*(3-15*r5+12*r5^2)*(3*q5-
7.5*q5^2+4*q5^3)+((1/s.^3)*(1.5*r5^2-2.5*r5^3+r5^4)*(24*q5-15))),7)
vymax = vpa(delta1*Q*a,7)
%slope
r6 = input('Enter the value for slopex r3:');
q6 = input('Enter the value for slopex q3:');
r7 = input('Enter the value for slopey r4:');
q7 = input('Enter the value for slopey q4:');
yr = vpa((u*(3*r6-7.5*r6^2+4*r6^3)*(1.5*q6^2-2.5*q6^3+q6^4))/D,7)
slopex = vpa(yr*Q*a^3,5)
yq = vpa((u*(1.5*r7^2-2.5*r7^3+r7^4)*(3*q7-7.5*q7^2+4*q7^3))/(s*D),7)
slopey = vpa(yq*Q*a^3,7)
%Bulkling Load or Resistant of the plate
n = vpa((Y1+(2*Y3/s^2)+(Y2/s^4))/Y5,7)
%in term Nx = n1*D/b^2
n1 = n*s^2
n2 = n1/pi^2
Nx = vpa((n*D/a^2),7)
%Fundamental Natural frequency
f = vpa(sqrt((Y1+(2*Y3/s^2)+(Y2/s^4))/Y6),7)
```

```
f1 = f/pi^2
F = vpa((f/a^2)*sqrt(D/p*h),7)
```

3. 3.1(vi) CCCS Plate

```
clc
%PROGRAM FOR CCCS PLATE
syms r q
U = 1.5*r^2-2.5*r^3+r^4;
V = q^2-2*q^3+q^4;
diff(U,2);
(diff(U,2))^2;
y1 = int((diff(U,2))^2,r,0,1);
z1 = int(V^2,q,0,1);
Y1 = y1*z1;
diff(V,2);
(diff(V,2))^2;
y2 = int(U^2,r,0,1);
z2 = int((diff(V,2))^2,q,0,1);
Y2 = y2*z2;
diff(U,1);
diff(V,1);
y3 = int((diff(U,1))^2,r,0,1);
z3 = int((diff(V,1))^2,q,0,1);
Y3 = y3*z3;
y4 = int(U,r,0,1);
z4 = int(V,q,0,1);
Y4 = y4*z4;
z5 = int(V^2,q,0,1);
Y5 = y3*z5;
Y6 = y2*z1;
v = input('Enter value of poission ratio v:');
a = input('Enter the horizontal dimension a(m):');
b = input('Enter the vertical dimension b(m):');
Q = input('Enter the udl Q(kN/m):');
```

```
h = input('Enter the thickness h(m):');
E = input('Enter the value of young modulus E:');
p = input('Enter the value of specific density p:');
echo on
s = b/a
echo off
u = vpa(Y4/(Y1+(2*Y3/s^2)+(Y2/s^4)),5)
%The flexural Rigidity of plate D is
D = E*h^3/(12*(1-v^2));
r = input('Enter value of r for deflection r:');
q = input('Enter value of q for deflection q:');
k = (1.5*r^2-2.5*r^3+r^4)*(q^2-2*q^3+q^4);
%Amplitude is
A = u*Q*a^4/D;
%deflection max
alpha = vpa((u*k),7)
Wmax = vpa((alpha*Q*a^4/D),7)
%Center Moment
r1 = input('Enter value of r for center moment r1:');
q1 = input('Enter value of q for center moment q1:');
beta = vpa(-u*((3-15*r1+12*r1^2)*(q1^2-2*q1^3+q1^4)+(v/s.^2)*(1.5*r1^2-
2.5*r1^3+r1^4)*(2-12*q1+12*q1^2)),5)
Mxc = vpa(beta*Q*a^2,5)
beta1 = vpa(-u*(v*(3-15*r1+12*r1^2)*(q1^2-2*q1^3+q1^4)+(1/s.^2)*(1.5*r1^2-
2.5*r1^3+r1^4)*(2-12*q1+12*q1^2)),5)
Myc = vpa(beta1*Q*a^2,5)
%Fixed edge moment
r2 = input('Enter the value for edge moment in X-D r2:');
q2 = input('Enter the value for edge moment in X-D q2:');
r3 = input('Enter the value for edge moment in Y-D r3:');
q3 = input('Enter the value for edge moment in Y-D q3:');
beta = vpa(-u*((3-15*r2+12*r2^2)*(q2^2-2*q2^3+q2^4)+(v/s.^2)*(1.5*r2^2-
2.5*r2^3+r2^4)*(2-12*q2+12*q2^2)),5)
Mxe = vpa(beta*Q*a^2,5)
```

```
beta1 = vpa(-u*(v*(3-15*r3+12*r3^2)*(q3^2-2*q3^3+q3^4)+(1/s.^2)*(1.5*r3^2-
2.5*r3^3+r3^4)*(2-12*q3+12*q3^2)),5)
Mye = vpa(beta1*Q*a^2,5)
%Shear Force
r4 = input('Enter the value for Vx r4:');
q4 = input('Enter the value for Vx q4:');
r5 = input('Enter the value for Vy r5:');
q5 = input('Enter the value for Vy q5:');
delta =vpa(-u*((24*r4-15)*(q4^2-2*q4^3+q4^4)+((2-v)/s.^2)*(3*r4-7.5*r4^2+4*r4^3)*(2-
12*r4+12*q4^2)),5)
Vx = vpa(delta*Q*a,5)
delta1 = vpa(-u*(((2-v)/s.^1)*(3-15*r5+12*r5^2)*(2*q5-
6*q5^2+4*q5^3)+((1/s.^3)*(1.5*r5^2-2.5*r5^3+r5^4)*(24*q5-12))),5)
Vy = vpa(delta1*Q*a,5)
%slope
r6 = input('Enter the value for slopex r6:');
q6 = input('Enter the value for slopex q6:');
r7 = input('Enter the value for slopey r7:');
q7 = input('Enter the value for slopey q7:');
yr = vpa((u*(3*r6-7.5*r6^2+4*r6^3)*(q6^2-2*q6^3+q6^4))/D,5)
Slopex = vpa(yr*Q*a^3,5)
yq = vpa((u*(1.5*r7^2-2.5*r7^3+r7^4)*(2*q7-6*q7^2+4*q7^3))/(s*D),5)
Slopey = vpa(yq*Q*a^3,5)
%Bulkling Load or Resistant of the plate
n = vpa((Y1+(2*Y3/s^2)+(Y2/s^4))/Y5,7)
%in term Nx = n1*D/b^2
n1 = n*s^2
n2 = n1/pi^2
Nx = vpa((n*D/a^2),7)
%Fundamental Natural frequency
f = vpa(sqrt((Y1+(2*Y3/s^2)+(Y2/s^4))/Y6),7)
f1 = f/pi^2
F = vpa((f/a^2)*sqrt(D/p*h),7)
```

3. 3.1(vii) SSFS Plate

```
clc
%PROGRAM FOR SSFS PLATE
syms r q
U = r-2*r^3+r^4;
V = 2.33*q-3.33*q^3+3.33*q^4-q^5;
diff(U,2);
(diff(U,2))^2;
y1 = int((diff(U,2))^2,r,0,1);
z1 = int(V^2,q,0,1);
Y1 = y1*z1;
diff(V,2);
(diff(V,2))^2;
y2 = int(U^2,r,0,1);
z2 = int((diff(V,2))^2,q,0,1);
Y2 = y2*z2;
diff(U,1);
diff(V,1);
y3 = int((diff(U,1))^2,r,0,1);
z3 = int((diff(V,1))^2,q,0,1);
Y3 = y3*z3;
y4 = int(U,r,0,1);
z4 = int(V,q,0,1);
Y4 = y4*z4;
z5 = int(V^2,q,0,1);
Y5 = y3*z5;
Y6 = y2*z1;
v = input('Enter value of poission ratio v:');
a = input('Enter the horizontal dimension a(m):');
b = input('Enter the vertical dimension b(m):');
Q = input('Enter the udl Q(kN/m):');
h = input('Enter the thickness h(m):');
E = input('Enter the value of young modulus E:');
p = input('Enter the value of specific density p:');
```

```
echo on
s = b/a
echo off
u = vpa(Y4/(Y1+(2*Y3/s^2)+(Y2/s^4)),5)
%The flexural Rigidity of plate D is
D = E*h^3/(12*(1-v^2));
r = input('Enter value of r for deflection r:');
q = input('Enter value of q for deflection q:');
k = (r-2*r^3+r^4)*(2.33*q-3.33*q^3+3.33*q^4-q^5);
%Amplitude is
A = u*Q*a^4/D;
%deflection max
alpha = vpa((u*k),5)
Wmax = vpa((alpha*Q*a^4/D),5)
%Center Moment
r1 = input('Enter value of r for center moment r1:');
q1 = input('Enter value of q for center moment q1:');
beta = vpa(-u*((12*r1^2-12*r1)*(2.33*q1-3.33*q1^3+3.33*q1^4-q1^5)+(v/s.^2)*(r1-
2*r1^3+r1^4)*(-20*q1+40*q1^2-20*q1^3)),5)
Mxc = vpa(beta*Q*a^2,5)
beta1 = vpa(-u*(v*(12*r1^2-12*r1)*(2.33*q1-3.33*q1^3+3.33*q1^4-q1^5)+(1/s.^2)*(r1-
2*r1^3+r1^4)*(-20*q1+40*q1^2-20*q1^3)),5)
Myc = vpa(beta1*Q*a^2,5)
%Fixed edge Moment
r2 = input('Enter the value for edge moment in X-D r2:');
q2 = input('Enter the value for edge moment in X-D q2:');
r3 = input('Enter the value for edge moment in Y-D r3:');
q3 = input('Enter the value for edge moment in Y-D q3:');
beta2 = vpa(-u*((12*r2^2-12*r2)*(2.33*q2-3.33*q2^3+3.33*q2^4-q2^5)+(v/s.^2)*(r2-
2*r2^3+r2^4)*(-20*q2+40*q2^2-20*q2^3)),5)
Mxe = vpa(beta2*Q*a^2,5)
beta3 = vpa(-u*(v*(12*r3^2-12*r3)*(2.33*q3-3.33*q3^3+3.33*q3^4-q3^5)+(1/s.^2)*(r3-
2*r3^3+r3^4)*(-20*q3+40*q3^2-20*q3^3)),5)
Mye = vpa(beta3*Q*a^2,5)
```

```
%Shear Force
r4 = input('Enter the value for Vx r4:');
q4 = input('Enter the value for Vx q4:');
r5 = input('Enter the value for Vy r5:');
q5 = input('Enter the value for Vy q5:');
delta =vpa(-u*((24*r4-12)*(2.33*q4-3.33*q4^3+3.33*q4^4-q4^5)+((2-v)/s.^2)*(1-
6*r4^2+4*r4^3)*(-20*q4+40*q4^2-20*q4^3)),5)
Vx = vpa(delta*Q*a,5)
delta1 = vpa(-u*(((2-v)/s.^1)*(12*r5^2-12*r5)*(2.33-10*q5^2+13.33*q5^3-
5*q5^4)+((1/s.^3)*(r5-2*r5^3+r5^4)*(-20+80*q5-60*q5^2))),5)
Vy = vpa(delta1*Q*a,5)
%slope
r6 = input('Enter the value for slopex r6:');
q6 = input('Enter the value for slopex q6:');
r7 = input('Enter the value for slopey r7:');
q7 = input('Enter the value for slopey q7:');
yr = vpa((u*(1-6*r6^2+4*r6^3)*(2.33*q6-3.33*q6^3+3.33*q6^4-q6^5))/D,5)
slopex = vpa(yr*Q*a^3,5)
yq = vpa((u*(r7-2*r7^3+r7^4)*(2.33-10*q7^2+13.33*q7^3-5*q7^4))/(s*D),5)
Slopey = vpa(yq*Q*a^3,5)
%Bulkling Load or Resistant of the plate
n = vpa((Y1+(2*Y3/s^2)+(Y2/s^4))/Y5,7)
%in term Nx = n1*D/b^2
n1 = n*s^2
n2 = n1/pi^2
Nx = vpa((n*D/a^2),7)
%Fundamental Natural frequency
f = vpa(sqrt((Y1+(2*Y3/s^2)+(Y2/s^4))/Y6),7)
f1 = f/pi^2
F = vpa((f/a^2)*sqrt(D/p*h),7)
```

3. 3.1(viii) SCFS Plate

```
clc
%PROGRAM FOR SCFS PLATE
```

```
syms r q
U = 1.5*r^2-2.5*r^3+r^4;
V = 2.33*q-3.33*q^3+3.33*q^4-q^5;
diff(U,2);
(diff(U,2))^2;
y1 = int((diff(U,2))^2,r,0,1);
z1 = int(V^2,q,0,1);
Y1 = y1*z1;
diff(V,2);
(diff(V,2))^2;
y2 = int(U^2,r,0,1);
z2 = int((diff(V,2))^2,q,0,1);
Y2 = y2*z2;
diff(U,1);
diff(V,1);
y3 = int((diff(U,1))^2,r,0,1);
z3 = int((diff(V,1))^2,q,0,1);
Y3 = y3*z3;
y4 = int(U,r,0,1);
z4 = int(V,q,0,1);
Y4 = y4*z4;
z5 = int(V^2,q,0,1);
Y5 = y3*z5;
Y6 = y2*z1;
v = input('Enter value of poission ratio v:');
a = input('Enter the horizontal dimension a(m):');
b = input('Enter the vertical dimension b(m):');
Q = input('Enter the udl Q(kN/m):');
h = input('Enter the thickness h(m):');
E = input('Enter the value of young modulus E:');
p = input('Enter the value of specific density p:');
echo on
s = b/a
echo off
```

```
u = vpa(Y4/(Y1+(2*Y3/s^2)+(Y2/s^4)),5)
%The flexural Rigidity of plate D is
D = E*h^3/(12*(1-v^2));
r = input('Enter value of r for deflection r:');
q = input('Enter value of q for deflection q:');
k = (1.5*r^2-2.5*r^3+r^4)*(2.33*q-3.33*q^3+3.33*q^4-q^5);
%Amplitude is
A = u*Q*a^4/D;
%deflection max
alpha = vpa((u*k),5)
Wmax = vpa((alpha*Q*a^4/D),5)
%Center Moment
r1 = input('Enter value of r for center moment r1:');
q1 = input('Enter value of q for center moment q1:');
beta = vpa(-u*((12*r1^2-15*r1+3)*(2.33*q1-3.33*q1^3+3.33*q1^4-
q1^5)+(v/s.^2)*(1.5*r1^2-2.5*r1^3+r1^4)*(-20*q1+40*q1^2-20*q1^3)),5)
Mxc = vpa(beta*Q*a^2,5)
beta1 = vpa(-u*(v*(12*r1^2-15*r1+3)*(2.33*q1-3.33*q1^3+3.33*q1^4-
q1^5)+(1/s.^2)*(1.5*r1^2-2.5*r1^3+r1^4)*(-20*q1+40*q1^2-20*q1^3)),5)
Myc = vpa(beta1*Q*a^2,5)
%Fixed edge Moment
r2 = input('Enter the value for edge moment in X-D r2:');
q2 = input('Enter the value for edge moment in X-D q2:');
r3 = input('Enter the value for edge moment in Y-D r3:');
q3 = input('Enter the value for edge moment in Y-D q3:');
beta2 = vpa(-u*((12*r2^2-15*r2+3)*(2.33*q2-3.33*q2^3+3.33*q2^4-
q2^5)+(v/s.^2)*(1.5*r2^2-2.5*r2^3+r2^4)*(-20*q2+40*q2^2-20*q2^3)),5)
Mxe = vpa(beta2*Q*a^2,5)
beta3 = vpa(-u*(v*(12*r3^2-15*r3+3)*(2.33*q3-3.33*q3^3+3.33*q3^4-
q3^5)+(1/s.^2)*(1.5*r3^2-2.5*r3^3+r3^4)*(-20*q3+40*q3^2-20*q3^3)),5)
Mye = vpa(beta3*Q*a^2,5)
%Shear Force
r4 = input('Enter the value for Vx r4:');
q4 = input('Enter the value for Vx q4:');
```

```
r5 = input('Enter the value for Vy r5:');
q5 = input('Enter the value for Vy q5:');
delta =vpa(-u*((24*r4-15)*(2.33*q4-3.33*q4^3+3.33*q4^4-q4^5)+((2-v)/s.^2)*(3*r4-
7.5*r4^2+4*r4^3)*(-20*q4+40*q4^2-20*q4^3)),5)
Vx = vpa(delta*Q*a,5)
delta1 = vpa(-u*((((2-v)/s.^1)*(3-15*r5+12*r5^2)*(2.33-10*q5^2+13.33*q5^3-
5*q5^4)+((1/s.^3)*(1.5*r5^2-2.5*r5^3+r5^4)*(-20+80*q5-60*q5^2))),5)
Vy = vpa(delta1*Q*a,5)
%slope
r6 = input('Enter the value for slopex r6:');
q6 = input('Enter the value for slopex q6:');
r7 = input('Enter the value for slopey r7:');
q7 = input('Enter the value for slopey q7:');
yr = vpa((u*(3-15*r6+12*r6^2)*(2.33*q6-3.33*q6^3+3.33*q6^4-q6^5))/D,5)
slopex = vpa(yr*Q*a^3,5)
yq = vpa((u*(1.5*r7^7-2.5*r7^3+r7^4)*(2.33-10*q7^2+13.33*q7^3-5*q7^4))/(s*D),5)
Slopey = vpa(yq*Q*a^3,5)
%Bulkling Load or Resistant of the plate
n = vpa((Y1+(2*Y3/s^2)+(Y2/s^4))/Y5,7)
%in term Nx = n1*D/b^2
n1 = n*s^2
n2 = n1/pi^2
Nx = vpa((n*D/a^2),7)
%Fundamental Natural frequency
f = vpa(sqrt((Y1+(2*Y3/s^2)+(Y2/s^4))/Y6),7)
f1 = f/pi^2
F = vpa((f/a^2)*sqrt(D/p*h),7)
```

3. 3.1(ix) CSFS Plate

```
clc
%PROGRAM FOR CSFS PLATE
syms r q
U = r-2*r^3+r^4;
```

```
V = 2.8*q^2-5.2*q^3+3.8*q^4-q^5;
diff(U,2);
(diff(U,2))^2;
y1 = int((diff(U,2))^2,r,0,1);
z1 = int(V^2,q,0,1);
Y1 = y1*z1;
diff(V,2);
(diff(V,2))^2;
y2 = int(U^2,r,0,1);
z2 = int((diff(V,2))^2,q,0,1);
Y2 = y2*z2;
diff(U,1);
diff(V,1);
y3 = int((diff(U,1))^2,r,0,1);
z3 = int((diff(V,1))^2,q,0,1);
Y3 = y3*z3;
y4 = int(U,r,0,1);
z4 = int(V,q,0,1);
Y4 = y4*z4;
z5 = int(V^2,q,0,1);
Y5 = y3*z5;
Y6 = y2*z1;
v = input('Enter value of poission ratio v:');
a = input('Enter the horizontal dimension a(m):');
b = input('Enter the vertical dimension b(m):');
Q = input('Enter the udl Q(kN/m):');
h = input('Enter the thickness h(m):');
E = input('Enter the value of young modulus E:');
p = input('Enter the value of specific density p:');
echo on
s = b/a
echo off
u = vpa(Y4/(Y1+(2*Y3/s^2)+(Y2/s^4)),5)
%The flexural Rigidity of plate D is
```

```
D = E*h^3/(12*(1-v^2));
r = input('Enter value of r for deflection r:');
q = input('Enter value of q for deflection q:');
k = (r-2*r^3+r^4)*(2.8*q^2-5.2*q^3+3.8*q^4-q^5);
%Amplitude is
A = u*Q*a^4/D;
%deflection max
alpha = vpa((u*k),5)
Wmax = vpa((alpha*Q*a^4/D),5)
%Center Moment
r1 = input('Enter value of r for center moment r1:');
q1 = input('Enter value of q for center moment q1:');
beta = vpa(-u*((12*r1^2-12*r1)*(2.8*q1^2-5.2*q1^3+3.8*q1^4-q1^5)+(v/s.^2)*(r1-
2*r1^3+r1^4)*(5.6-31.2*q1+45.6*q1^2-20*q1^3)),5)
Mxc = vpa(beta*Q*a^2,5)
beta1 = vpa(-u*((v*(-12*r1+12*r1^2)*(2.8*q1^2-5.2*q1^3+3.8*q1^4-q1^5))+(1/s.^2)*(r1-
2*r1^3+r1^4)*(5.6-31.2*q1+45.6*q1^2-20*q1^3)),5)
myc = vpa(beta1*Q*a^2,5)
%Fixed edge moment
r2 = input('Enter the value for edge moment in X-D r2:');
q2 = input('Enter the value for edge moment in X-D q2:');
r3 = input('Enter the value for edge moment in Y-D r3:');
q3 = input('Enter the value for edge moment in Y-D q3:');
beta2 = vpa(-u*((-12*r2+12*r2^2)*(2.8*q2^2-5.2*q2^3+3.8*q2^4-q2^5)+(v/s.^2)*(r2-
2*r2^3+r2^4)*(5.6-31.2*q2+45.6*q2^2-20*q2^3)),5)
Mxe = vpa(beta2*Q*a^2,5)
beta3 = vpa(-u*((v*(-12*r3+12*r3^2)*(2.8*q3^2-5.2*q3^3+3.8*q3^4-q3^5))+(1/s.^2)*(r3-
2*r3^3+r3^4)*(5.6-31.2*q3+45.6*q3^2-20*q3^3)),5)
Mye = vpa(beta3*Q*a^2,5)
%Shear Force
r4 = input('Enter the value for Vx r4:');
q4 = input('Enter the value for Vx q4:');
r5 = input('Enter the value for Vy r5:');
q5 = input('Enter the value for Vy q5:');
```

```
delta =vpa(-u*((24*r4-12)*(2.8*q4^2-5.2*q4^3+3.8*q4^4-q4^5)+((2-v)/s^3)*(1-
6*r4^2+4*r4^3)*(5.6-31.2*q4+45.6*q4^2-20*q4^3)),5)
Vx = vpa(delta*Q*a,5)
delta1 = vpa(-u*((((2-v)/s.^1)*(-12*r5+12*r5^2)*(5.6*r5-15.6*q5^2+15.2*q5^3-
5*q5^4)+(1/s.^3)*(r5-2*r5^3+r5^4)*(-31.2+91.2*q5-60*q5^2)),5)
Vy = vpa(delta1*Q*a,5)
%slope
r6 = input('Enter the value for slopex r6:');
q6 = input('Enter the value for slopex q6:');
r7 = input('Enter the value for slopey r7:');
q7 = input('Enter the value for slopey q7:');
yr = vpa((u*(1-6*r6^2+4*r6^3)*(2.8*q6^2-5.2*q6^3+3.8*q6^4-q6^5))/D,5)
slopex = vpa(yr*Q*a^3,5)
yq = vpa((u*(r7-2*r7^3+r7^4)*(5.6*r7-15.6*q7^2+15.2*q7^3-5*q7^4))/(s*D),5)
Slopey = vpa(yq*Q*a^3,5)
%Bulkling Load or Resistant of the plate
n = vpa((Y1+(2*Y3/s^2)+(Y2/s^4))/Y5,7)
%in term Nx = n1*D/b^2
n1 = n*s^2
n2 = n1/pi^2
Nx = vpa((n*D/a^2),7)
%Fundamental Natural frequency
f = vpa(sqrt((Y1+(2*Y3/s^2)+(Y2/s^4))/Y6),7)
f1 = f/pi^2
F = vpa((f/a^2)*sqrt(D/p*h),7)
```

3. 3.1(x) CCFS Plate

```
clc
%PROGRAM FOR CCFS PLATE
syms r q
U = 1.5*r^2-2.5*r^3+r^4;
V = 2.8*q^2-5.2*q^3+3.8*q^4-q^5;
diff(U,2);
(diff(U,2))^2;
```

```
y1 = int((diff(U,2))^2,r,0,1);
z1 = int(V^2,q,0,1);
Y1 = y1*z1;
diff(V,2);
(diff(V,2))^2;
y2 = int(U^2,r,0,1);
z2 = int((diff(V,2))^2,q,0,1);
Y2 = y2*z2;
diff(U,1);
diff(V,1);
y3 = int((diff(U,1))^2,r,0,1);
z3 = int((diff(V,1))^2,q,0,1);
Y3 = y3*z3;
y4 = int(U,r,0,1);
z4 = int(V,q,0,1);
Y4 = y4*z4;
z5 = int(V^2,q,0,1);
Y5 = y3*z5;
Y6 = y2*z1;
v = input('Enter value of poission ratio v:');
a = input('Enter the horizontal dimension a(m):');
b = input('Enter the vertical dimension b(m):');
Q = input('Enter the udl Q(kN/m):');
h = input('Enter the thickness h(m):');
E = input('Enter the value of young modulus E:');
p = input('Enter the value of specific density p:');
echo on
s = b/a
echo off
u = vpa(Y4/(Y1+(2*Y3/s^2)+(Y2/s^4)),5)
%The flexural Rigidity of plate D is
D = E*h^3/(12*(1-v^2));
r = input('Enter value of r for deflection r:');
q = input('Enter value of q for deflection q:');
```

```
k = (1.5*r^2-2.5*r^3+r^4)*(2.8*q^2-5.2*q^3+3.8*q^4-q^5);
%Amplitude is
A = u*Q*a^4/D;
%deflection max
alpha = vpa((u*k),5)
Wmax = vpa((alpha*Q*a^4/D),5)
%Center Moment
r1 = input('Enter value of r for center moment r1:');
q1 = input('Enter value of q for center moment q1:');
beta = vpa(-u*((3-15*r1+12*r1^2)*(2.8*q1^2-5.2*q1^3+3.8*q1^4-
q1^5)+(v/s.^2)*(1.5*r1^2-2.5*r1^3+r1^4)*(5.6-31.2*q1+45.6*q1^2-20*q1^3)),5)
Mxc = vpa(beta*Q*a^2,5)
beta1 = vpa(-u*((v*(3-15*r1+12*r1^2)*(2.8*q1^2-5.2*q1^3+3.8*q1^4-
q1^5))+(1/s.^2)*(1.5*r1^2-2.5*r1^3+r1^4)*(5.6-31.2*q1+45.6*q1^2-20*q1^3)),5)
myc = vpa(beta1*Q*a^2,5)
%Fixed edge moment
r2 = input('Enter the value for edge moment in X-D r2:');
q2 = input('Enter the value for edge moment in X-D q2:');
r3 = input('Enter the value for edge moment in Y-D r3:');
q3 = input('Enter the value for edge moment in Y-D q3:');
beta2 = vpa(-u*((3-15*r2+12*r2^2)*(2.8*q2^2-5.2*q2^3+3.8*q2^4-
q2^5)+(v/s.^2)*(1.5*r2^2-2.5*r2^3+r2^4)*(5.6-31.2*q2+45.6*q2^2-20*q2^3)),5)
Mxe = vpa(beta2*Q*a^2,5)
beta3 = vpa(-u*((v*(3-15*r3+12*r3^2)*(2.8*q3^2-5.2*q3^3+3.8*q3^4-
q3^5))+(1/s.^2)*(1.5*r3^2-2.5*r3^3+r3^4)*(5.6-31.2*q3+45.6*q3^2-20*q3^3)),5)
Mye = vpa(beta3*Q*a^2,5)
%Shear Force
r4 = input('Enter the value for Vx r4:');
q4 = input('Enter the value for Vx q4:');
r5 = input('Enter the value for Vy r5:');
q5 = input('Enter the value for Vy q5:');
delta =vpa(-u*((24*r4-15)*(2.8*q4^2-5.2*q4^3+3.8*q4^4-q4^5)+((2-v)/s^3)*(3*r4-
7.5*r4^2+4*r4^3)*(5.6-31.2*q4+45.6*q4^2-20*q4^3)),5)
Vx = vpa(delta*Q*a,5)
```

```
delta1 = vpa(-u*(((2-v)/s.^1)*(3-15*r5+12*r5^2)*(5.6*r5-15.6*q5^2+15.2*q5^3-
5*q5^4)+(1/s.^3)*(1.5*r5^2-2.5*r5^3+r5^4)*(-31.2+91.2*q5-60*q5^2)),5)
Vy = vpa(delta1*Q*a,5)
%slope
r6 = input('Enter the value for slopex r6:');
q6 = input('Enter the value for slopex q6:');
r7 = input('Enter the value for slopey r7:');
q7 = input('Enter the value for slopey q7:');
yr = vpa((u*(3*r6-7.5*r6^2+4*r6^3)*(2.8*q6^2-5.2*q6^3+3.8*q6^4-q6^5))/D,5)
slopex = vpa(yr*Q*a^3,5)
yq = vpa((u*(1.5*r7^2-2.5*r7^3+r7^4)*(5.6*r7-15.6*q7^2+15.2*q7^3-5*q7^4))/(s*D),5)
Slopey = vpa(yq*Q*a^3,5)
%Bulkling Load or Resistant of the plate
n = vpa((Y1+(2*Y3/s^2)+(Y2/s^4))/Y5,7)
%in term Nx = n1*D/b^2
n1 = n*s^2
n2 = n1/pi^2
Nx = vpa((n*D/a^2),7)
%Fundamental Natural frequency
f = vpa(sqrt((Y1+(2*Y3/s^2)+(Y2/s^4))/Y6),7)
f1 = f/pi^2
F = vpa((f/a^2)*sqrt(D/p*h),7)
```

3. 3.1(xi) SCFC Plate

```
clc
%PROGRAM FOR SCFC PLATE
syms r q
U = r^2-2*r^3+r^4;
V = 2.33*q-3.33*q^3+3.33*q^4-q^5;
diff(U,2);
(diff(U,2))^2;
y1 = int((diff(U,2))^2,r,0,1);
z1 = int(V^2,q,0,1);
Y1 = y1*z1;
```

```
diff(V,2);
(diff(V,2))^2;
y2 = int(U^2,r,0,1);
z2 = int((diff(V,2))^2,q,0,1);
Y2 = y2*z2;
diff(U,1);
diff(V,1);
y3 = int((diff(U,1))^2,r,0,1);
z3 = int((diff(V,1))^2,q,0,1);
Y3 = y3*z3;
y4 = int(U,r,0,1);
z4 = int(V,q,0,1);
Y4 = y4*z4;
z5 = int(V^2,q,0,1);
Y5 = y3*z5;
Y6 = y2*z1;
v = input('Enter value of poission ratio v:');
a = input('Enter the horizontal dimension a(m):');
b = input('Enter the vertical dimension b(m):');
Q = input('Enter the udl Q(kN/m):');
h = input('Enter the thickness h(m):');
E = input('Enter the value of young modulus E:');
p = input('Enter the value of specific density p:');
echo on
s = b/a
echo off
u = vpa(Y4/(Y1+(2*Y3/s^2)+(Y2/s^4)),5)
%The flexural Rigidity of plate D is
D = E*h^3/(12*(1-v^2));
r = input('Enter value of r for deflection r:');
q = input('Enter value of q for deflection q:');
k = (r^2-2*r^3+r^4)*(2.33*q-3.33*q^3+3.33*q^4-q^5);
%Amplitude is
A = u*Q*a^4/D;
```

```
%deflection max
alpha = vpa((u*k),5)
Wmax = vpa((alpha*Q*a^4/D),5)
%Center Moment
r1 = input('Enter value of r for center moment r1:');
q1 = input('Enter value of q for center moment q1:');
beta = vpa(-u*((12*r1^2-12*r1+2)*(2.33*q1-3.33*q1^3+3.33*q1^4-q1^5)+(v/s.^2)*(r1^2-
2*r1^3+r1^4)*(-20*q1+40*q1^2-20*q1^3)),5)
Mxc = vpa(beta*Q*a^2,5)
beta1 = vpa(-u*(v*(12*r1^2-12*r1+2)*(2.33*q1-3.33*q1^3+3.33*q1^4-
q1^5)+(1/s.^2)*(r1^2-2*r1^3+r1^4)*(-20*q1+40*q1^2-20*q1^3)),5)
Myc = vpa(beta1*Q*a^2,5)
%Fixed edge Moment
r2 = input('Enter the value for edge moment in X-D r2:');
q2 = input('Enter the value for edge moment in X-D q2:');
r3 = input('Enter the value for edge moment in Y-D r3:');
q3 = input('Enter the value for edge moment in Y-D q3:');
beta2 = vpa(-u*((12*r2^2-12*r2+2)*(2.33*q2-3.33*q2^3+3.33*q2^4-q2^5)+(v/s.^2)*(r2^2-
2*r2^3+r2^4)*(-20*q2+40*q2^2-20*q2^3)),5)
Mxe = vpa(beta2*Q*a^2,5)
beta3 = vpa(-u*(v*(12*r3^2-12*r3+2)*(2.33*q3-3.33*q3^3+3.33*q3^4-
q3^5)+(1/s.^2)*(r3^2-2*r3^3+r3^4)*(-20*q3+40*q3^2-20*q3^3)),5)
Mye = vpa(beta3*Q*a^2,5)
%Shear Force
r4 = input('Enter the value for Vx r4:');
q4 = input('Enter the value for Vx q4:');
r5 = input('Enter the value for Vy r5:');
q5 = input('Enter the value for Vy q5:');
delta =vpa(-u*((24*r4-12)*(2.33*q4-3.33*q4^3+3.33*q4^4-q4^5)+((2-v)/s.^2)*(2*r4-
6*r4^2+4*r4^3)*(-20*q4+40*q4^2-20*q4^3)),5)
Vx = vpa(delta*Q*a,5)
delta1 = vpa(-u*(((2-v)/s.^1)*(2-12*r5+12*r5^2)*(2.33-10*q5^2+13.33*q5^3-
5*q5^4)+((1/s.^3)*(r5^2-2*r5^3+r5^4)*(-20+80*q5-60*q5^2))),5)
Vy = vpa(delta1*Q*a,5)
```

```
%slope
r6 = input('Enter the value for slopex r6:');
q6 = input('Enter the value for slopex q6:');
r7 = input('Enter the value for slopey r7:');
q7 = input('Enter the value for slopey q7:');
yr = vpa((u*(2*r6-6*r6+12*r6^2)*(2.33*q6-3.33*q6^3+3.33*q6^4-q6^5))/D,5)
slopex = vpa(yr*Q*a^3,5)
yq = vpa((u*(r7^7-2*r7^3+r7^4)*(2.33-10*q7^2+13.33*q7^3-5*q7^4))/(s*D),5)
Slopey = vpa(yq*Q*a^3,5)
%Bulkling Load or Resistant of the plate
n = vpa((Y1+(2*Y3/s^2)+(Y2/s^4))/Y5,7)
%in term Nx = n1*D/b^2
n1 = n*s^2
n2 = n1/pi^2
Nx = vpa((n*D/a^2),7)
%Fundamental Natural frequency
f = vpa(sqrt((Y1+(2*Y3/s^2)+(Y2/s^4))/Y6),7)
f1 = f/pi^2
F = vpa((f/a^2)*sqrt(D/p*h),7)
```

3. 3.1(xii) CCFC Plate

```
clc
%PROGRAM FOR CCFC PLATE
syms r q
U = r^2-2*r^3+r^4;
V = 2.8*q^2-5.2*q^3+3.8*q^4-q^5;
diff(U,2);
(diff(U,2))^2;
y1 = int((diff(U,2))^2,r,0,1);
z1 = int(V^2,q,0,1);
Y1 = y1*z1;
diff(V,2);
(diff(V,2))^2;
y2 = int(U^2,r,0,1);
```

```
z2 = int((diff(V,2))^2,q,0,1);
Y2 = y2*z2;
diff(U,1);
diff(V,1);
y3 = int((diff(U,1))^2,r,0,1);
z3 = int((diff(V,1))^2,q,0,1);
Y3 = y3*z3;
y4 = int(U,r,0,1);
z4 = int(V,q,0,1);
Y4 = y4*z4;
z5 = int(V^2,q,0,1);
Y5 = y3*z5;
Y6 = y2*z1;
v = input('Enter value of poission ratio v:');
a = input('Enter the horizontal dimension a(m):');
b = input('Enter the vertical dimension b(m):');
Q = input('Enter the udl Q(kN/m):');
h = input('Enter the thickness h(m):');
E = input('Enter the value of young modulus E:');
p = input('Enter the value of specific density p:');
echo on
s = b/a
echo off
u = vpa(Y4/(Y1+(2*Y3/s^2)+(Y2/s^4)),5)
%The flexural Rigidity of plate D is
D = E*h^3/(12*(1-v^2));
r = input('Enter value of r for deflection r:');
q = input('Enter value of q for deflection q:');
k = (r^2-2*r^3+r^4)*(2.8*q^2-5.2*q^3+3.8*q^4-q^5);
%Amplitude is
A = u*Q*a^4/D;
%deflection max
alpha = vpa((u*k),5)
Wmax = vpa((alpha*Q*a^4/D),5)
```

```
%Center Moment
r1 = input('Enter value of r for center moment r1:');
q1 = input('Enter value of q for center moment q1:');
beta = vpa(-u*((2-12*r1+12*r1^2)*(2.8*q1^2-5.2*q1^3+3.8*q1^4-q1^5)+(v/s.^2)*(r1^2-
2*r1^3+r1^4)*(5.6-31.2*q1+45.6*q1^2-20*q1^3)),5)
Mxc = vpa(beta*Q*a^2,5)
beta1 = vpa(-u*((v*(2-12*r1+12*r1^2)*(2.8*q1^2-5.2*q1^3+3.8*q1^4-
q1^5))+(1/s.^2)*(r1^2-2*r1^3+r1^4)*(5.6-31.2*q1+45.6*q1^2-20*q1^3)),5)
myc = vpa(beta1*Q*a^2,5)
%Fixed edge moment
r2 = input('Enter the value for edge moment in X-D r2:');
q2 = input('Enter the value for edge moment in X-D q2:');
r3 = input('Enter the value for edge moment in Y-D r3:');
q3 = input('Enter the value for edge moment in Y-D q3:');
beta2 = vpa(-u*((2-12*r2+12*r2^2)*(2.8*q2^2-5.2*q2^3+3.8*q2^4-q2^5)+(v/s.^2)*(r2^2-
2*r2^3+r2^4)*(5.6-31.2*q2+45.6*q2^2-20*q2^3)),5)
Mxe = vpa(beta2*Q*a^2,5)
beta3 = vpa(-u*((v*(2-12*r3+12*r3^2)*(2.8*q3^2-5.2*q3^3+3.8*q3^4-
q3^5))+(1/s.^2)*(r3^2-2*r3^3+r3^4)*(5.6-31.2*q3+45.6*q3^2-20*q3^3)),5)
Mye = vpa(beta3*Q*a^2,5)
%Shear Force
r4 = input('Enter the value for Vx r4:');
q4 = input('Enter the value for Vx q4:');
r5 = input('Enter the value for Vy r5:');
q5 = input('Enter the value for Vy q5:');
delta =vpa(-u*((24*r4-12)*(2.8*q4^2-5.2*q4^3+3.8*q4^4-q4^5)+((2-v)/s^3)*(2*r4-
6*r4^2+4*r4^3)*(5.6-31.2*q4+45.6*q4^2-20*q4^3)),5)
Vx = vpa(delta*Q*a,5)
delta1 = vpa(-u*((((2-v)/s.^1)*(2-12*r5+12*r5^2)*(5.6*r5-15.6*q5^2+15.2*q5^3-
5*q5^4)+(1/s.^3)*(r5^2-2*r5^3+r5^4)*(-31.2+91.2*q5-60*q5^2)),5)
Vy = vpa(delta1*Q*a,5)
%slope
r6 = input('Enter the value for slopex r6:');
q6 = input('Enter the value for slopex q6:');
```

```
r7 = input('Enter the value for slopey r7:');
q7 = input('Enter the value for slopey q7:');
yr = vpa((u*(2*r6-6*r6^2+4*r6^3)*(2.8*q6^2-5.2*q6^3+3.8*q6^4-q6^5))/D,5)
slopex = vpa(yr*Q*a^3,5)
yq = vpa((u*(r7^2-2*r7^3+r7^4)*(5.6*r7-15.6*q7^2+15.2*q7^3-5*q7^4))/(s*D),5)
Slopey = vpa(yq*Q*a^3,5)
%Bulkling Load or Resistant of the plate
n = vpa((Y1+(2*Y3/s^2)+(Y2/s^4))/Y5,7)
%in term Nx = n1*D/b^2
n1 = n*s^2
n2 = n1/pi^2
Nx = vpa((n*D/a^2),7)
%Fundamental Natural frequency
f = vpa(sqrt((Y1+(2*Y3/s^2)+(Y2/s^4))/Y6),7)
f1 = f/pi^2
F = vpa((f/a^2)*sqrt(D/p*h),7)
```

APPENDIX 2

ONE-WAY CONTINUOUS PLATE PROGRAM

The program to analyze this plate is presented below:

```
clear all
%CONTINUOUS PLATE IN ONE DIRECTION (X-DIRECTION) PROGRAM
a = input('Enter the horizontal dimension a(m):');
b = input('Enter the vertical dimension b(m):');
Q = input('Enter the udl Q(kN/m):');
v = input('Enter poision ratio v:');
echo on
s = b/a
echo off
%for SSSC span1
syms r q
U = 0.5*r-1.5*r^3+r^4;
V = q-2*q^3+q^4;
diff(U,2);
(diff(U,2))^2;
y1 = int((diff(U,2))^2,r,0,1);
z1 = int(V^2,q,0,1);
Y1 = y1*z1;
diff(V,2);
(diff(V,2))^2;
y2 = int(U^2,r,0,1);
z2 = int((diff(V,2))^2,q,0,1);
```

```
Y2 = y2*z2;
diff(U,1);
diff(V,1);
y3 = int((diff(U,1))^2,r,0,1);
z3 = int((diff(V,1))^2,q,0,1);
Y3 = y3*z3;
y4 = int(U,r,0,1);
z4 = int(V,q,0,1);
Y4 = y4*z4;
u = vpa(Y4/(Y1+(2*Y3/s^2)+(Y2/s^4)),5);
r = input('Enter r value at support 2 r:');
q = input('Enter q value at support 2 q:');
k = (0.5*r-1.5*r^3+r^4)*(q-2*q^3+q^4);
echo on
%Edge moment
echo off
beta = vpa(-u*((12*r^2-9*r)*(q-2*q^3+q^4)+(v/s.^2)*(0.5*r-1.5*r^3+r^4)*(12*q^2-
12*q)),5);
mxe = vpa(beta*Q*a^2,5)
%for SCSC span2
syms r q
U = r^2-2*r^3+r^4;
V = q-2*q^3+q^4;
diff(U,2);
(diff(U,2))^2;
y1 = int((diff(U,2))^2,r,0,1);
z1 = int(V^2,q,0,1);
Y1 = y1*z1;
diff(V,2);
(diff(V,2))^2;
y2 = int(U^2,r,0,1);
z2 = int((diff(V,2))^2,q,0,1);
Y2 = y2*z2;
diff(U,1);
```

```
diff(V,1);
y3 = int((diff(U,1))^2,r,0,1);
z3 = int((diff(V,1))^2,q,0,1);
Y3 = y3*z3;
y4 = int(U,r,0,1);
z4 = int(V,q,0,1);
Y4 = y4*z4;
u1 = vpa(Y4/(Y1+(2*Y3/s^2)+(Y2/s^4)),5);
r1 = input('Enter r1 value at support 2 r1:');
q1 = input('Enter q1 value at support 2 q1:');
k1 = (r1^2-2*r1^3+r1^4)*(q1-2*q1^3+q1^4);
echo on
%Edge moment
echo off
beta1 = vpa(-u1*((2-12*r1+12*r1^2)*(q1-2*q1^3+q1^4)+(v/s.^2)*(r1^2-
2*r1^3+r1^4)*(12*q1^2-12*q1)),5);
mxe1a = vpa(beta1*Q*a^2,5)
mxe1b = mxe1a
%for SCSC span3
echo on
%Edge moment
echo off
mxe2a = mxe1a
mxe2b = mxe1b
%for SCSS span4
syms r q
U = 1.5*r^2-2.5*r^3+r^4;
V = q-2*q^3+q^4;
diff(U,2);
(diff(U,2))^2;
y1 = int((diff(U,2))^2,r,0,1);
z1 = int(V^2,q,0,1);
Y1 = y1*z1;
diff(V,2);
```

```
(diff(V,2))^2;
y2 = int(U^2,r,0,1);
z2 = int((diff(V,2))^2,q,0,1);
Y2 = y2*z2;
diff(U,1);
diff(V,1);
y3 = int((diff(U,1))^2,r,0,1);
z3 = int((diff(V,1))^2,q,0,1);
Y3 = y3*z3;
y4 = int(U,r,0,1);
z4 = int(V,q,0,1);
Y4 = y4*z4;
u3 = vpa(Y4/(Y1+(2*Y3/s^2)+(Y2/s^4)),5)
r3 = input('Enter r3 value at support 4 r3:');
q3 = input('Enter q3 value at support 4 q3:');
k3 = (1.5*r3^2-2.5*r3^3+r3^4)*(q3-2*q3^3+q3^4);
echo on
%Edge moment
echo off
beta3 = vpa(-u3*((3-15*r3+12*r3^2)*(q3-2*q3^3+q3^4)+(v/s.^2)*(1.5*r3^2-
2*r3^3+r3^4)*(12*q3^2-12*q3)),5);
mxe3 = vpa(beta3*Q*a^2,5)
echo on
%Fixed end moment at each support(starting from left to right of the plate span)
echo off
f1 = 0
f2 = mxe-mxe1a
f3 = mxe1b-mxe2a
f4 = mxe2b-mxe3
f5 = 0
%using element stiffness method of analysis
E = input('young modulus E:');
I = input('second moment of inertia I:')
%flexural rigidity (EI) = FR
```

```
FR = E*I;
A = [4 2; 2 4];
%element stiffness
K1 = A.*FR;
K2 = K1;
K3 = K1;
K4 = K1;
%global stiffness K
K = [4 2 0 0 0; 2 8 2 0 0; 0 2 8 2 0; 0 0 2 8 2; 0 0 0 2 4];
f = Q*a^2
B = [f1; f2; f3; f4; f5];
F = B.*f
echo on
%F = K*theta, hence, theta = inv(K)*F
%theta = [theta1; theta2; theta3; theta4; theta5]
echo off
theta = K\F
%For member forces(md)
%md = [md..; md..]
echo on
%enter the values of terms of theta matrix above.
echo off
theta1 = input('theta1:');
theta2 = input('theta2:');
theta3 = input('theta3:');
theta4 = input('theta4:');
theta5 = input('theta5:');
thetaa = [theta1; theta2];
thetab = [theta2; theta3];
thetac = [theta3; theta4];
thetad = [theta4; theta5];
md12 = K1*thetaa
md23 = K2*thetab
md34 = K3*thetac
```

```
md45 = K4*thetad
%Fixed end or edge moment
fem = [0; mxe; mxe1a; mxe1b; mxe2a; mxe2b; mxe3; 0];
echo on
%Final Support moment(m)
echo off
%m = fem-md, m12 =
fem12 = [0; mxe];
fem23 = [mxe1a; mxe1b];
fem34 = [mxe2a; mxe2b];
fem45 = [mxe3; 0];
m12 = fem12-md12
m23 = fem23-md23
m34 = fem34-md34
m45 = fem45-md45
%Average final Support monent(mavs) is mavs... = (m21+m22)/2
echo on
%input the positive values of m matrix above. note: m12 = [ms11;ms21], m23 = [m22;m32]
...
echo off
ms11 = input('Enter monent at support 1 member 1:');
ms21 = input('Enter monent at support 2 member 1:');
ms22 = input('Enter monent at support 2 member 2:');
ms32 = input('Enter monent at support 3 member 2:');
ms33 = input('Enter monent at support 3 member 3:');
ms43 = input('Enter monent at support 4 member 3:');
ms44 = input('Enter monent at support 4 member 4:');
ms54 = input('Enter monent at support 5 member 4:');
echo on
%average final support moment
echo off
mavs1 = ms11
mavs2 = (ms21+ms22)*0.5
mavs3 = (ms32+ms33)*0.5
```

```
mavs4 = (ms43+ms44)*0.5
mavs5 = ms54
echo on
%Span Moment(mspan)
echo off
%mspan = 0.125*Q*a^2-(mst+mst)*o.5
c = 0.125*Q*a^2;
mspan12 = c-((mavs1+mavs2)*0.5)
mspan23 = c-((mavs2+mavs3)*0.5)
mspan34 = c-((mavs3+mavs4)*0.5)
mspan45 = c-((mavs4+mavs5)*0.5)
R = 0:0.125:1;
bm = [mavs1 -mspan12 mavs2 -mspan23 mavs3 -mspan34 mavs4 -mspan45 mavs5];
plot(R,bm,'r-')
grid
xlabel('R');
ylabel('bm');
title('BMD for Single Continuous plate')
```

APPENDIX 3

TWO-WAY CONTINUOUS PLATE PROGRAM

The program to analyze this plate is as follows:

```
clear all
%CONTINUOUS PLATE IN TWO DIRECTION STRIP S-S IN X-DIRECTION
PROGRAM
a = input('Enter the horizontal dimension a(m):');
b = input('Enter the vertical dimension b(m):');
Q = input('Enter the udl Q(kN/m):');
v = input('Enter poision ratio v:');
echo on
s = b/a
echo off
%for SSSC span1
syms r q
U = 0.5*r-1.5*r^3+r^4;
V = 0.5*q-1.5*q^3+q^4;
diff(U,2);
(diff(U,2))^2;
y1 = int((diff(U,2))^2,r,0,1);
z1 = int(V^2,q,0,1);
Y1 = y1*z1;
diff(V,2);
(diff(V,2))^2;
y2 = int(U^2,r,0,1);
```

```
z2 = int((diff(V,2))^2,q,0,1);
Y2 = y2*z2;
diff(U,1);
diff(V,1);
y3 = int((diff(U,1))^2,r,0,1);
z3 = int((diff(V,1))^2,q,0,1);
Y3 = y3*z3;
y4 = int(U,r,0,1);
z4 = int(V,q,0,1);
Y4 = y4*z4;
u = vpa(Y4/(Y1+(2*Y3/s^2)+(Y2/s^4)),5);
r = input('Enter r value at support 2 r:');
q = input('Enter q value at support 2 q:');
k = (0.5*r-1.5*r^3+r^4)*(0.5*q-1.5*q^3+q^4);
echo on
%Edge moment
echo off
beta = vpa(-u*((12*r^2-9*r)*(0.5*q-1.5*q^3+q^4)+(v/s.^2)*(0.5*r-1.5*r^3+r^4)*(12*q^2-
9*q)),5);
mxe = vpa(beta*Q*a^2,5)
%for SCSC span2
syms r q
U = r^2-2*r^3+r^4;
V = 0.5*q-1.5*q^3+q^4;
diff(U,2);
(diff(U,2))^2;
y1 = int((diff(U,2))^2,r,0,1);
z1 = int(V^2,q,0,1);
Y1 = y1*z1;
diff(V,2);
(diff(V,2))^2;
y2 = int(U^2,r,0,1);
z2 = int((diff(V,2))^2,q,0,1);
Y2 = y2*z2;
```

```
diff(U,1);
diff(V,1);
y3 = int((diff(U,1))^2,r,0,1);
z3 = int((diff(V,1))^2,q,0,1);
Y3 = y3*z3;
y4 = int(U,r,0,1);
z4 = int(V,q,0,1);
Y4 = y4*z4;
u1 = vpa(Y4/(Y1+(2*Y3/s^2)+(Y2/s^4)),5);
r1 = input('Enter r1 value at support 2 r1:');
q1 = input('Enter q1 value at support 2 q1:');
k1 = (r1^2-2*r1^3+r1^4)*(0.5*q1-1.5*q1^3+q1^4);
echo on
%Edge moment
echo off
beta1 = vpa(-u1*((2-12*r1+12*r1^2)*(0.5*q1-1.5*q1^3+q1^4)+(v/s.^2)*(r1^2-
2*r1^3+r1^4)*(12*q1^2-9*q1)),5);
mxe1a = vpa(beta1*Q*a^2,5)
mxe1b = mxe1a
%for SCSC span3
echo on
%Edge moment
echo off
mxe2a = mxe1a
mxe2b = mxe1b
%for SCSS span4
syms r q
U = 1.5*r^2-2.5*r^3+r^4;
V = 0.5*q-1.5*q^3+q^4;
diff(U,2);
(diff(U,2))^2;
y1 = int((diff(U,2))^2,r,0,1);
z1 = int(V^2,q,0,1);
Y1 = y1*z1;
```

```
diff(V,2);
(diff(V,2))^2;
y2 = int(U^2,r,0,1);
z2 = int((diff(V,2))^2,q,0,1);
Y2 = y2*z2;
diff(U,1);
diff(V,1);
y3 = int((diff(U,1))^2,r,0,1);
z3 = int((diff(V,1))^2,q,0,1);
Y3 = y3*z3;
y4 = int(U,r,0,1);
z4 = int(V,q,0,1);
Y4 = y4*z4;
u3 = vpa(Y4/(Y1+(2*Y3/s^2)+(Y2/s^4)),5)
r3 = input('Enter r3 value at support 4 r3:');
q3 = input('Enter q3 value at support 4 q3:');
k3 = (1.5*r3^2-2.5*r3^3+r3^4)*(0.5*q3-1.5*q3^3+q3^4);
echo on
%Edge moment
echo off
beta3 = vpa(-u3*((3-15*r3+12*r3^2)*(0.5*q3-1.5*q3^3+q3^4)+(v/s.^2)*(1.5*r3^2-
2.5*r3^3+r3^4)*(12*q3^2-9*q3)),5);
mxe3 = vpa(beta3*Q*a^2,5)
echo on
%Fixed end moment at each support(starting from left to right of the plate span)
echo off
f1 = 0
f2 = mxe-mxe1a
f3 = mxe1b-mxe2a
f4 = mxe2b-mxe3
f5 = 0
%using element stiffness method of analysis
E = input('young modulus E:');
I = input('second moment of inertia I:')
```

```
%flexural rigidity (EI) = FR
FR = E*I;
A = [4 2; 2 4];
%element stiffness
K1 = A.*FR;
K2 = K1;
K3 = K1;
K4 = K1;
%global stiffness K
K = [4 2 0 0 0; 2 8 2 0 0; 0 2 8 2 0; 0 0 2 8 2; 0 0 0 2 4];
f = Q*a^2
B = [f1; f2; f3; f4; f5];
F = B.*f
echo on
%F = K*theta, hence, theta = inv(K)*F
%theta = [theta1; theta2; theta3; theta4; theta5]
echo off
theta = K\F
%For member forces(md)
%md = [md..; md..]
echo on
%enter the valuues of terms of theta matrix above.
echo off
theta1 = input('theta1:');
theta2 = input('theta2:');
theta3 = input('theta3:');
theta4 = input('theta4:');
theta5 = input('theta5:');
thetaa = [theta1; theta2];
thetab = [theta2; theta3];
thetac = [theta3; theta4];
thetad = [theta4; theta5];
md12 = K1*thetaa
md23 = K2*thetab
```

```
md34 = K3*thetac
md45 = K4*thetad
%Fixed end or edge moment
fem = [0; mxe; mxe1a; mxe1b; mxe2a; mxe2b; mxe3; 0];
echo on
%Final Support moment(m)
echo off
%m = fem-md, m12 =
fem12 = [0; mxe];
fem23 = [mxe1a; mxe1b];
fem34 = [mxe2a; mxe2b];
fem45 = [mxe3; 0];
m12 = fem12-md12
m23 = fem23-md23
m34 = fem34-md34
m45 = fem45-md45
%Average final Support monent(mavs) is mavs... = (m21+m22)/2
echo on
%input the positive values of m matrix above. note: m12 = [ms11;ms21], m23 = [m22;m32]
echo off
ms11 = input('Enter monent at support 1 member 1:');
ms21 = input('Enter monent at support 2 member 1:');
ms22 = input('Enter monent at support 2 member 2:');
ms32 = input('Enter monent at support 3 member 2:');
ms33 = input('Enter monent at support 3 member 3:');
ms43 = input('Enter monent at support 4 member 3:');
ms44 = input('Enter monent at support 4 member 4:');
ms54 = input('Enter monent at support 5 member 4:');
echo on
%average final support moment
echo off
mavs1 = ms11
mavs2 = (ms21+ms22)*0.5
mavs3 = (ms32+ms33)*0.5
```

```
mavs4 = (ms43+ms44)*0.5
mavs5 = ms54
echo on
%Span Moment(mspan)
echo off
%mspan = 0.125*Q*a^2-(mst+mst)*o.5
c = 0.125*Q*a^2;
mspan12 = c-((mavs1+mavs2)*0.5)
mspan23 = c-((mavs2+mavs3)*0.5)
mspan34 = c-((mavs3+mavs4)*0.5)
mspan45 = c-((mavs4+mavs5)*0.5)
R = 0:0.125:1;
bm = [mavs1 -mspan12 mavs2 -mspan23 mavs3 -mspan34 mavs4 -mspan45 mavs5];
plot(R,bm,'r-')
grid
xlabel('R');
ylabel('bm');
title('BMD for Strip S-S 2way Continuous plate')
hold on
echo on
%CONTINUOUS PLATE IN TWO DIRECTION STRIP T-T IN X-DIRECTION
PROGRAM
echo off
%for CSCC span1
syms r q
U = 0.5*r-1.5*r^3+r^4;
V = q^2-2*q^3+q^4;
diff(U,2);
(diff(U,2))^2;
y1 = int((diff(U,2))^2,r,0,1);
z1 = int(V^2,q,0,1);
Y1 = y1*z1;
diff(V,2);
(diff(V,2))^2;
```

```
y2 = int(U^2,r,0,1);
z2 = int((diff(V,2))^2,q,0,1);
Y2 = y2*z2;
diff(U,1);
diff(V,1);
y3 = int((diff(U,1))^2,r,0,1);
z3 = int((diff(V,1))^2,q,0,1);
Y3 = y3*z3;
y4 = int(U,r,0,1);
z4 = int(V,q,0,1);
Y4 = y4*z4;
u = vpa(Y4/(Y1+(2*Y3/s^2)+(Y2/s^4)),5);
r = input('Enter r value at support 2 r:');
q = input('Enter q value at support 2 q:');
k = (0.5*r-1.5*r^3+r^4)*(q^2-2*q^3+q^4);
echo on
%Edge moment
echo off
beta = vpa(-u*((12*r^2-9*r)*(q^2-2*q^3+q^4)+(v/s.^2)*(0.5*r-1.5*r^3+r^4)*(2-12*q-
12*q^2)),5);
mxe = vpa(beta*Q*a^2,5)
%for scsc span2
syms r q
U = r^2-2*r^3+r^4;
V = q^2-2*q^3+q^4;
diff(U,2);
(diff(U,2))^2;
y1 = int((diff(U,2))^2,r,0,1);
z1 = int(V^2,q,0,1);
Y1 = y1*z1;
diff(V,2);
(diff(V,2))^2;
y2 = int(U^2,r,0,1);
z2 = int((diff(V,2))^2,q,0,1);
```

```
Y2 = y2*z2;
diff(U,1);
diff(V,1);
y3 = int((diff(U,1))^2,r,0,1);
z3 = int((diff(V,1))^2,q,0,1);
Y3 = y3*z3;
y4 = int(U,r,0,1);
z4 = int(V,q,0,1);
Y4 = y4*z4;
u1 = vpa(Y4/(Y1+(2*Y3/s^2)+(Y2/s^4)),5);
r1 = input('Enter r1 value at support 2 r1:');
q1 = input('Enter q1 value at support 2 q1:');
k1 = (r1^2-2*r1^3+r1^4)*(q1^2-2*q1^3+q1^4);
echo on
%Edge moment
echo off
beta1 = vpa(-u1*((2-12*r1+12*r1^2)*(q1^2-2*q1^3+q1^4)+(v/s.^2)*(r1^2-2*r1^3+r1^4)*(2-
12*q1+12*q1^2)),5);
mxe1a = vpa(beta1*Q*a^2,5)
mxe1b = mxe1a
%for scsc span3
echo on
%Edge moment
echo off
mxe2a = mxe1a
mxe2b = mxe1b
%for scss span4
syms r q
U = 1.5*r^2-2.5*r^3+r^4;
V = q^2-2*q^3+q^4;
diff(U,2);
(diff(U,2))^2;
y1 = int((diff(U,2))^2,r,0,1);
z1 = int(V^2,q,0,1);
```

```
Y1 = y1*z1;
diff(V,2);
(diff(V,2))^2;
y2 = int(U^2,r,0,1);
z2 = int((diff(V,2))^2,q,0,1);
Y2 = y2*z2;
diff(U,1);
diff(V,1);
y3 = int((diff(U,1))^2,r,0,1);
z3 = int((diff(V,1))^2,q,0,1);
Y3 = y3*z3;
y4 = int(U,r,0,1);
z4 = int(V,q,0,1);
Y4 = y4*z4;
u3 = vpa(Y4/(Y1+(2*Y3/s^2)+(Y2/s^4)),5)
r3 = input('Enter r3 value at support 4 r3:');
q3 = input('Enter q3 value at support 4 q3:');
k3 = (1.5*r3^2-2.5*r3^3+r3^4)*(q3^2-2*q3^3+q3^4);
echo on
%Edge moment
echo off
beta3 = vpa(-u3*((3-15*r3+12*r3^2)*(q3^2-2*q3^3+q3^4)+(v/s.^2)*(1.5*r3^2-
2*r3^3+r3^4)*(2-12*q3+12*q3^2)),5);
mxe3 = vpa(beta3*Q*a^2,5)
echo on
%Fixed end moment at each support(starting from left to right of the plate span)
echo off
f1 = 0
f2 = mxe-mxe1a
f3 = mxe1b-mxe2a
f4 = mxe2b-mxe3
f5 = 0
%using element stiffness method of analysis
E = input('young modulus E:');
```

```
I = input('second moment of inertia I:')
%flexural rigidity (EI) = FR
FR = E*I;
A = [4 2; 2 4];
%element stiffness
K1 = A.*FR;
K2 = K1;
K3 = K1;
K4 = K1;
%global stiffness K
K = [4 2 0 0 0; 2 8 2 0 0; 0 2 8 2 0; 0 0 2 8 2; 0 0 0 2 4];
f = Q*a^2
B = [f1; f2; f3; f4; f5];
F = B.*f
echo on
%F = K*theta, hence, theta = inv(K)*F
%theta = [theta1; theta2; theta3; theta4; theta5]
echo off
theta = K\F
%For member forces(md)
%md = [md..; md..]
echo on
%enter the valuues of terms of theta matrix above.
echo off
theta1 = input('theta1:');
theta2 = input('theta2:');
theta3 = input('theta3:');
theta4 = input('theta4:');
theta5 = input('theta5:');
thetaa = [theta1; theta2];
thetab = [theta2; theta3];
thetac = [theta3; theta4];
thetad = [theta4; theta5];
md12 = K1*thetaa
```

```
md23 = K2*thetab
md34 = K3*thetac
md45 = K4*thetad
%Fixed end or edge moment
fem = [0; mxe; mxe1a; mxe1b; mxe2a; mxe2b; mxe3; 0];
echo on
%Final Support moment(m)
echo off
%m = fem-md, m12 =
fem12 = [0; mxe];
fem23 = [mxe1a; mxe1b];
fem34 = [mxe2a; mxe2b];
fem45 = [mxe3; 0];
m12 = fem12-md12
m23 = fem23-md23
m34 = fem34-md34
m45 = fem45-md45
%Average final Support monent(mavs) is mavs... = (m21+m22)/2
echo on
%input the positive values of m matrix above. note: m12 = [ms11;ms21], m23 = [m22;m32]
...
echo off
ms11 = input('Enter monent at support 1 member 1:');
ms21 = input('Enter monent at support 2 member 1:');
ms22 = input('Enter monent at support 2 member 2:');
ms32 = input('Enter monent at support 3 member 2:');
ms33 = input('Enter monent at support 3 member 3:');
ms43 = input('Enter monent at support 4 member 3:');
ms44 = input('Enter monent at support 4 member 4:');
ms54 = input('Enter monent at support 5 member 4:');
echo on
%average final support moment
echo off
mavs1 = ms11
```

```
mavs2 = (ms21+ms22)*0.5
mavs3 = (ms32+ms33)*0.5
mavs4 = (ms43+ms44)*0.5
mavs5 = ms54
echo on
%Span Moment(mspan)
echo off
%mspan = 0.125*Q*a^2-(mst+mst)*o.5
c = 0.125*Q*a^2;
mspan12 = c-((mavs1+mavs2)*0.5)
mspan23 = c-((mavs2+mavs3)*0.5)
mspan34 = c-((mavs3+mavs4)*0.5)
mspan45 = c-((mavs4+mavs5)*0.5)
R = 0:0.125:1;
bm = [mavs1 -mspan12 mavs2 -mspan23 mavs3 -mspan34 mavs4 -mspan45 mavs5];
plot(R,bm,'r-')
grid
xlabel('R');
ylabel('bm');
title('BMD for strip T-T 2way Continuous plate')
hold on
echo on
%CONTINUOUS PLATE IN TWO DIRECTION STRIP Sb-Sb IN X-DIRECTION
PROGRAM
echo off
%for CSSC span1
syms r q
U = 0.5*r-1.5*r^3+r^4;
V = 1.5*q^2-2.5*q^3+q^4;
diff(U,2);
(diff(U,2))^2;
y1 = int((diff(U,2))^2,r,0,1);
z1 = int(V^2,q,0,1);
Y1 = y1*z1;
```

```
diff(V,2);
(diff(V,2))^2;
y2 = int(U^2,r,0,1);
z2 = int((diff(V,2))^2,q,0,1);
Y2 = y2*z2;
diff(U,1);
diff(V,1);
y3 = int((diff(U,1))^2,r,0,1);
z3 = int((diff(V,1))^2,q,0,1);
Y3 = y3*z3;
y4 = int(U,r,0,1);
z4 = int(V,q,0,1);
Y4 = y4*z4;
u = vpa(Y4/(Y1+(2*Y3/s^2)+(Y2/s^4)),5);
r = input('Enter r value at support 2 r:');
q = input('Enter q value at support 2 q:');
k = (0.5*r-1.5*r^3+r^4)*(1.5*q^2-2.5*q^3+q^4);
echo on
%Edge moment
echo off
beta = vpa(-u*((12*r^2-9*r)*(1.5*q^2-2.5*q^3+q^4)+(v/s.^2)*(0.5*r-1.5*r^3+r^4)*(3-
15*q+12*q^2)),5);
mxe = vpa(beta*Q*a^2,5)
%for CCSC span2
syms r q
U = r^2-2*r^3+r^4;
V = 1.5*q^2-2.5*q^3+q^4;
diff(U,2);
(diff(U,2))^2;
y1 = int((diff(U,2))^2,r,0,1);
z1 = int(V^2,q,0,1);
Y1 = y1*z1;
diff(V,2);
(diff(V,2))^2;
```

```
y2 = int(U^2,r,0,1);
z2 = int((diff(V,2))^2,q,0,1);
Y2 = y2*z2;
diff(U,1);
diff(V,1);
y3 = int((diff(U,1))^2,r,0,1);
z3 = int((diff(V,1))^2,q,0,1);
Y3 = y3*z3;
y4 = int(U,r,0,1);
z4 = int(V,q,0,1);
Y4 = y4*z4;
u1 = vpa(Y4/(Y1+(2*Y3/s^2)+(Y2/s^4)),5);
r1 = input('Enter r1 value at support 2 r1:');
q1 = input('Enter q1 value at support 2 q1:');
k1 = (r1^2-2*r1^3+r1^4)*(1.5*q1^2-2.5*q1^3+q1^4);
echo on
%Edge moment
echo off
beta1 = vpa(-u1*((2-12*r1+12*r1^2)*(1.5*q1^2-2.5*q1^3+q1^4)+(v/s.^2)*(r1^2-
2*r1^3+r1^4)*(3-15*q1+12*q1^2)),5);
mxe1a = vpa(beta1*Q*a^2,5)
mxe1b = mxe1a
%for CCSC span3
echo on
%Edge moment
echo off
mxe2a = mxe1a
mxe2b = mxe1b
%for CCSS span4
syms r q
U = 1.5*r^2-2.5*r^3+r^4;
V = 1.5*q^2-2.5*q^3+q^4;
diff(U,2);
(diff(U,2))^2;
```

```
y1 = int((diff(U,2))^2,r,0,1);
z1 = int(V^2,q,0,1);
Y1 = y1*z1;
diff(V,2);
(diff(V,2))^2;
y2 = int(U^2,r,0,1);
z2 = int((diff(V,2))^2,q,0,1);
Y2 = y2*z2;
diff(U,1);
diff(V,1);
y3 = int((diff(U,1))^2,r,0,1);
z3 = int((diff(V,1))^2,q,0,1);
Y3 = y3*z3;
y4 = int(U,r,0,1);
z4 = int(V,q,0,1);
Y4 = y4*z4;
u3 = vpa(Y4/(Y1+(2*Y3/s^2)+(Y2/s^4)),5)
r3 = input('Enter r3 value at support 4 r3:');
q3 = input('Enter q3 value at support 4 q3:');
k3 = (1.5*r3^2-2.5*r3^3+r3^4)*(1.5*q3^2-2.5*q3^3+q3^4);
echo on
%Edge moment
echo off
beta3 = vpa(-u3*((3-15*r3+12*r3^2)*(1.5*q3^2-2.5*q3^3+q3^4)+(v/s.^2)*(1.5*r3^2-
2*r3^3+r3^4)*(3-15*q3+12*q3^2)),5);
mxe3 = vpa(beta3*Q*a^2,5)
echo on
%Fixed end moment at each support(starting from left to right of the plate span)
echo off
f1 = 0
f2 = mxe-mxe1a
f3 = mxe1b-mxe2a
f4 = mxe2b-mxe3
f5 = 0
```

```
%using element stiffness method of analysis
E = input('young modulus E:');
I = input('second moment of inertia I:')
%flexural rigidity (EI) = FR
FR = E*I;
A = [4 2; 2 4];
%element stiffness
K1 = A.*FR;
K2 = K1;
K3 = K1;
K4 = K1;
%global stiffness K
K = [4 2 0 0 0; 2 8 2 0 0; 0 2 8 2 0; 0 0 2 8 2; 0 0 0 2 4];
f = Q*a^2
B = [f1; f2; f3; f4; f5];
F = B.*f
echo on
%F = K*theta, hence, theta = inv(K)*F
%theta = [theta1; theta2; theta3; theta4; theta5]
echo off
theta = K\F
%For member forces(md)
%md = [md..; md..]
echo on
%enter the valuues of terms of theta matrix above.
echo off
theta1 = input('theta1:');
theta2 = input('theta2:');
theta3 = input('theta3:');
theta4 = input('theta4:');
theta5 = input('theta5:');
thetaa = [theta1; theta2];
thetab = [theta2; theta3];
thetac = [theta3; theta4];
```

```
thetad = [theta4; theta5];
md12 = K1*thetaa
md23 = K2*thetab
md34 = K3*thetac
md45 = K4*thetad
%Fixed end or edge moment
fem = [0; mxe; mxe1a; mxe1b; mxe2a; mxe2b; mxe3; 0];
echo on
%Final Support moment(m)
echo off
%m = fem-md, m12 =
fem12 = [0; mxe];
fem23 = [mxe1a; mxe1b];
fem34 = [mxe2a; mxe2b];
fem45 = [mxe3; 0];
m12 = fem12-md12
m23 = fem23-md23
m34 = fem34-md34
m45 = fem45-md45
%Average final Support monent(mavs) is mavs... = (m21+m22)/2
echo on
%input the positive values of m matrix above. note: m12 = [ms11;ms21], m23 = [m22;m32]
...
echo off
ms11 = input('Enter monent at support 1 member 1:');
ms21 = input('Enter monent at support 2 member 1:');
ms22 = input('Enter monent at support 2 member 2:');
ms32 = input('Enter monent at support 3 member 2:');
ms33 = input('Enter monent at support 3 member 3:');
ms43 = input('Enter monent at support 4 member 3:');
ms44 = input('Enter monent at support 4 member 4:');
ms54 = input('Enter monent at support 5 member 4:');
echo on
%average final support moment
```

```
echo off
mavs1 = ms11
mavs2 = (ms21+ms22)*0.5
mavs3 = (ms32+ms33)*0.5
mavs4 = (ms43+ms44)*0.5
mavs5 = ms54
echo on
%Span Moment(mspan)
echo off
%mspan = 0.125*Q*a^2-(mst+mst)*o.5
c = 0.125*Q*a^2;
mspan12 = c-((mavs1+mavs2)*0.5)
mspan23 = c-((mavs2+mavs3)*0.5)
mspan34 = c-((mavs3+mavs4)*0.5)
mspan45 = c-((mavs4+mavs5)*0.5)
R = 0:0.125:1;
bm = [mavs1 -mspan12 mavs2 -mspan23 mavs3 -mspan34 mavs4 -mspan45 mavs5];
plot(R,bm,'r-')
grid
xlabel('R');
ylabel('bm');
title('BMD for Single Continuous plate')
hold on
echo on
%CONTINUOUS PLATE IN TWO DIRECTION STRIP 1-1 IN Y-DIRECTION
PROGRAM
echo off
%for SSCC span1
syms r q
U = 0.5*r-1.5*r^3+r^4;
V = 0.5*q-1.5*q^3+q^4;
diff(U,2);
(diff(U,2))^2;
y1 = int((diff(U,2))^2,r,0,1);
```

```
z1 = int(V^2,q,0,1);
Y1 = y1*z1;
diff(V,2);
(diff(V,2))^2;
y2 = int(U^2,r,0,1);
z2 = int((diff(V,2))^2,q,0,1);
Y2 = y2*z2;
diff(U,1);
diff(V,1);
y3 = int((diff(U,1))^2,r,0,1);
z3 = int((diff(V,1))^2,q,0,1);
Y3 = y3*z3;
y4 = int(U,r,0,1);
z4 = int(V,q,0,1);
Y4 = y4*z4;
u = vpa(Y4/(Y1+(2*Y3/s^2)+(Y2/s^4)),5);
r = input('Enter r value at support 2 r:');
q = input('Enter q value at support 2 q:');
k = (0.5*r-1.5*r^3+r^4)*(0.5*q-1.5*q^3+q^4);
echo on
%Edge moment
echo off
beta = vpa(-u*(v*(12*r^2-9*r)*(0.5*q-1.5*q^3+q^4)+(1/s.^2)*(0.5*r-1.5*r^3+r^4)*(12*q^2-
9*q)),5);
mxe = vpa(beta*Q*a^2,5)
%for CSCC span2
syms r q
U = 0.5*r-1.5*r^3+r^4;
V = q^2-2*q^3+q^4;
diff(U,2);
(diff(U,2))^2;
y1 = int((diff(U,2))^2,r,0,1);
z1 = int(V^2,q,0,1);
Y1 = y1*z1;
```

```
diff(V,2);
(diff(V,2))^2;
y2 = int(U^2,r,0,1);
z2 = int((diff(V,2))^2,q,0,1);
Y2 = y2*z2;
diff(U,1);
diff(V,1);
y3 = int((diff(U,1))^2,r,0,1);
z3 = int((diff(V,1))^2,q,0,1);
Y3 = y3*z3;
y4 = int(U,r,0,1);
z4 = int(V,q,0,1);
Y4 = y4*z4;
u1 = vpa(Y4/(Y1+(2*Y3/s^2)+(Y2/s^4)),5);
r1 = input('Enter r1 value at support 2 r1:');
q1 = input('Enter q1 value at support 2 q1:');
k1 = (0.5*r1-1.5*r1^3+r1^4)*(q1^2-2*q1^3+q1^4);
echo on
%Edge moment
echo off
beta1 = vpa(-u1*(v*(12*r1^2-9*r1)*(q1^2-2*q1^3+q1^4)+(1/s.^2)*(0.5*r1-
1.5*r1^3+r1^4)*(2-12*q1+12*q1^2)),5);
mxe1a = vpa(beta1*Q*a^2,5)
mxe1b = mxe1a
%for CSSC span3
syms r q
U = 0.5*r-1.5*r^3+r^4;
V = 1.5*q^2-2.5*q^3+q^4;
diff(U,2);
(diff(U,2))^2;
y1 = int((diff(U,2))^2,r,0,1);
z1 = int(V^2,q,0,1);
Y1 = y1*z1;
diff(V,2);
```

```
(diff(V,2))^2;
y2 = int(U^2,r,0,1);
z2 = int((diff(V,2))^2,q,0,1);
Y2 = y2*z2;
diff(U,1);
diff(V,1);
y3 = int((diff(U,1))^2,r,0,1);
z3 = int((diff(V,1))^2,q,0,1);
Y3 = y3*z3;
y4 = int(U,r,0,1);
z4 = int(V,q,0,1);
Y4 = y4*z4;
u2 = vpa(Y4/(Y1+(2*Y3/s^2)+(Y2/s^4)),5)
r2 = input('Enter r2 value at support 4 r2:');
q2 = input('Enter q2 value at support 4 q2:');
k2 = (0.5*r2-1.5*r2^3+r2^4)*(1.5*q2^2-2.5*q2^3+q2^4);
echo on
%Edge moment
echo off
beta2 = vpa(-u2*(v*(12*r2^2-9*r2)*(1.5*q2^2-2.5*q2^3+q2^4)+(1/s.^2)*(0.5*r2-
1.5*r2^3+r2^4)*(3-15*q2+12*q2^2)),5);
mxe2a = vpa(beta2*Q*a^2,5)
echo on
%Fixed end moment at each support(starting from left to right of the plate span)
echo off
f1 = 0
f2 = mxe-mxe1a
f3 = mxe1b-mxe2a
f4 = 0
%using element stiffness method of analysis
E = input('young modulus E:');
I = input('second moment of inertia I:')
%flexural rigidity (EI) = FR
FR = E*I;
```

```
A = [4 2; 2 4];
%element stiffness
K1 = A.*FR;
K2 = K1;
K3 = K1;
%global stiffness K
K = [4 2 0 0 ; 2 8 2 0 ; 0 2 8 2 ; 0 0 2 4];
f = Q*a^2
B = [f1; f2; f3; f4];
F = B.*f
echo on
%F = K*theta, hence, theta = inv(K)*F
%theta = [theta1; theta2; theta3; theta4]
echo off
theta = K\F
%For member forces(md)
%md = [md..; md..]
echo on
%enter the valuues of terms of theta matrix above.
echo off
theta1 = input('theta1:');
theta2 = input('theta2:');
theta3 = input('theta3:');
theta4 = input('theta4:');
thetaa = [theta1; theta2];
thetab = [theta2; theta3];
thetac = [theta3; theta4];
md12 = K1*thetaa
md23 = K2*thetab
md34 = K3*thetac
%Fixed end or edge moment
fem = [0; mxe; mxe1a; mxe1b; mxe2a; 0];
echo on
%Final Support moment(m)
```

```
echo off
%m = fem-md, m12 =
fem12 = [0; mxe];
fem23 = [mxe1a; mxe1b];
fem34 = [mxe2a; 0];
m12 = fem12-md12
m23 = fem23-md23
m34 = fem34-md34
%Average final Support monent(mavs) is mavs... = (m21+m22)/2
echo on
%input the positive values of m matrix above. note: m12 = [ms11;ms21], m23 = [m22;m32]
...
echo off
ms11 = input('Enter monent at support 1 member 1:');
ms21 = input('Enter monent at support 2 member 1:');
ms22 = input('Enter monent at support 2 member 2:');
ms32 = input('Enter monent at support 3 member 2:');
ms33 = input('Enter monent at support 3 member 3:');
ms43 = input('Enter monent at support 4 member 3:');
echo on
%average final support moment
echo off
mavs1 = ms11
mavs2 = (ms21+ms22)*0.5
mavs3 = (ms32+ms33)*0.5
mavs4 = ms43
echo on
%Span Moment(mspan)
echo off
%mspan = 0.125*Q*a^2-(mst+mst)*o.5
c = 0.125*Q*a^2;
mspan12 = c-((mavs1+mavs2)*0.5)
mspan23 = c-((mavs2+mavs3)*0.5)
mspan34 = c-((mavs3+mavs4)*0.5)
```

```
R = 0:0.166666666:1;
bm = [mavs1 -mspan12 mavs2 -mspan23 mavs3 -mspan34 mavs4];
plot(R,bm,'r-')
grid
xlabel('R');
ylabel('bm');
title('BMD for strip 1-1 2way Continuous plate')
hold on
echo on
%CONTINUOUS PLATE IN TWO DIRECTION STRIP 2-2 IN Y-DIRECTION
PROGRAM
echo off
%for SCCC span1
syms r q
U = r^2-2*r^3+r^4;
V = 0.5*q-1.5*q^3+q^4;
diff(U,2);
(diff(U,2))^2;
y1 = int((diff(U,2))^2,r,0,1);
z1 = int(V^2,q,0,1);
Y1 = y1*z1;
diff(V,2);
(diff(V,2))^2;
y2 = int(U^2,r,0,1);
z2 = int((diff(V,2))^2,q,0,1);
Y2 = y2*z2;
diff(U,1);
diff(V,1);
y3 = int((diff(U,1))^2,r,0,1);
z3 = int((diff(V,1))^2,q,0,1);
Y3 = y3*z3;
y4 = int(U,r,0,1);
z4 = int(V,q,0,1);
Y4 = y4*z4;
```

```
u = vpa(Y4/(Y1+(2*Y3/s^2)+(Y2/s^4)),5);
r = input('Enter r value at support 2 r:');
q = input('Enter q value at support 2 q:');
k = (r^2-2*r^3+r^4)*(0.5*q-1.5*q^3+q^4);
echo on
%Edge moment
echo off
beta = vpa(-u*(v*(2-12*r+12*r^2)*(0.5*q-1.5*q^3+q^4)+(1/s.^2)*(r^2-2*r^3+r^4)*(12*q^2-
9*q)),5);
mxe = vpa(beta*Q*a^2,5)
%for CCCC span2
syms r q
U = r^2-2*r^3+r^4;
V = q^2-2*q^3+q^4;
diff(U,2);
(diff(U,2))^2;
y1 = int((diff(U,2))^2,r,0,1);
z1 = int(V^2,q,0,1);
Y1 = y1*z1;
diff(V,2);
(diff(V,2))^2;
y2 = int(U^2,r,0,1);
z2 = int((diff(V,2))^2,q,0,1);
Y2 = y2*z2;
diff(U,1);
diff(V,1);
y3 = int((diff(U,1))^2,r,0,1);
z3 = int((diff(V,1))^2,q,0,1);
Y3 = y3*z3;
y4 = int(U,r,0,1);
z4 = int(V,q,0,1);
Y4 = y4*z4;
u1 = vpa(Y4/(Y1+(2*Y3/s^2)+(Y2/s^4)),5);
r1 = input('Enter r1 value at support 2 r1:');
```

```
q1 = input('Enter q1 value at support 2 q1:');
k1 = (r1^2-2*r1^3+r1^4)*(q1^2-2*q1^3+q1^4);
echo on
%Edge moment
echo off
beta1 = vpa(-u1*(v*(2-12*r1+12*r1^2)*(q1^2-2*q1^3+q1^4)+(1/s.^2)*(r1^2-
2*r1^3+r1^4)*(2-12*q1+12*q1^2)),5);
mxe1a = vpa(beta1*Q*a^2,5)
mxe1b = mxe1a
%for CCSC span3
syms r q
U = r^2-2*r^3+r^4;
V = 1.5*q^2-2.5*q^3+q^4;
diff(U,2);
(diff(U,2))^2;
y1 = int((diff(U,2))^2,r,0,1);
z1 = int(V^2,q,0,1);
Y1 = y1*z1;
diff(V,2);
(diff(V,2))^2;
y2 = int(U^2,r,0,1);
z2 = int((diff(V,2))^2,q,0,1);
Y2 = y2*z2;
diff(U,1);
diff(V,1);
y3 = int((diff(U,1))^2,r,0,1);
z3 = int((diff(V,1))^2,q,0,1);
Y3 = y3*z3;
y4 = int(U,r,0,1);
z4 = int(V,q,0,1);
Y4 = y4*z4;
u2 = vpa(Y4/(Y1+(2*Y3/s^2)+(Y2/s^4)),5)
r2 = input('Enter r2 value at support 4 r2:');
q2 = input('Enter q2 value at support 4 q2:');
```

```
k2 = (r2^2-2*r2^3+r2^4)*(1.5*q2^2-2.5*q2^3+q2^4);
echo on
%Edge moment
echo off
beta2 = vpa(-u2*(v*(2-12*r2+12*r2^2)*(1.5*q2^2-2.5*q2^3+q2^4)+(1/s.^2)*(r2^2-
2*r2^3+r2^4)*(3-15*q2+12*q2^2)),5);
mxe2a = vpa(beta2*Q*a^2,5)
echo on
%Fixed end moment at each support(starting from left to right of the plate span)
echo off
f1 = 0
f2 = mxe-mxe1a
f3 = mxe1b-mxe2a
f4 = 0
%using element stiffness method of analysis
E = input('young modulus E:');
I = input('second moment of inertia I:')
%flexural rigidity (EI) = FR
FR = E*I;
A = [4 2; 2 4];
%element stiffness
K1 = A.*FR;
K2 = K1;
K3 = K1;
%global stiffness K
K = [4 2 0 0 ; 2 8 2 0 ; 0 2 8 2 ; 0 0 2 4];
f = Q*a^2
B = [f1; f2; f3; f4];
F = B.*f
echo on
%F = K*theta, hence, theta = inv(K)*F
%theta = [theta1; theta2; theta3; theta4]
echo off
theta = K\F
```

```
%For member forces(md)
%md = [md..; md..]
echo on
%enter the valuues of terms of theta matrix above.
echo off
theta1 = input('theta1:');
theta2 = input('theta2:');
theta3 = input('theta3:');
theta4 = input('theta4:');
thetaa = [theta1; theta2];
thetab = [theta2; theta3];
thetac = [theta3; theta4];
md12 = K1*thetaa
md23 = K2*thetab
md34 = K3*thetac
%Fixed end or edge moment
fem = [0; mxe; mxe1a; mxe1b; mxe2a; 0];
echo on
%Final Support moment(m)
echo off
%m = fem-md, m12 =
fem12 = [0; mxe];
fem23 = [mxe1a; mxe1b];
fem34 = [mxe2a; 0];
m12 = fem12-md12
m23 = fem23-md23
m34 = fem34-md34
%Average final Support monent(mavs) is mavs... = (m21+m22)/2
echo on
%input the positive values of m matrix above. note: m12 = [ms11;ms21], m23 = [m22;m32]
...
echo off
ms11 = input('Enter monent at support 1 member 1:');
ms21 = input('Enter monent at support 2 member 1:');
```

```
ms22 = input('Enter monent at support 2 member 2:');
ms32 = input('Enter monent at support 3 member 2:');
ms33 = input('Enter monent at support 3 member 3:');
ms43 = input('Enter monent at support 4 member 3:');
echo on
%average final support moment
echo off
mavs1 = ms11
mavs2 = (ms21ms22)*0.5
mavs3 = (ms32+ms33)*0.5
mavs4 = ms43
echo on
%Span Moment(mspan)
echo off
%mspan = 0.125*Q*a^2-(mst+mst)*o.5
c = 0.125*Q*a^2;
mspan12 = c-((mavs1+mavs2)*0.5)
mspan23 = c-((mavs2+mavs3)*0.5)
mspan34 = c-((mavs3+mavs4)*0.5)
R = 0:0.166666666:1;
bm = [mavs1 -mspan12 mavs2 -mspan23 mavs3 -mspan34 mavs4];
plot(R,bm,'r-')
grid
xlabel('R');
ylabel('bm');
title('BMD for Strip 2-2 & 2b-2b 2way Continuous plate')
echo on
%CONTINUOUS PLATE IN TWO DIRECTION STRIP 2b-2b IN Y-DIRECTION
PROGRAM
%strip 2-2 and strip 2b-2b) ie span 2 and 3 are the same.
echo off
echo on
%CONTINUOUS PLATE IN TWO DIRECTION STRIP 1b-1b IN Y-DIRECTION
PROGRAM
```

```
echo off
%for SCCS span1
syms r q
U = 1.5*r^2-2.5*r^3+r^4;
V = 0.5*q-1.5*q^3+q^4;
diff(U,2);
(diff(U,2))^2;
y1 = int((diff(U,2))^2,r,0,1);
z1 = int(V^2,q,0,1);
Y1 = y1*z1;
diff(V,2);
(diff(V,2))^2;
y2 = int(U^2,r,0,1);
z2 = int((diff(V,2))^2,q,0,1);
Y2 = y2*z2;
diff(U,1);
diff(V,1);
y3 = int((diff(U,1))^2,r,0,1);
z3 = int((diff(V,1))^2,q,0,1);
Y3 = y3*z3;
y4 = int(U,r,0,1);
z4 = int(V,q,0,1);
Y4 = y4*z4;
u = vpa(Y4/(Y1+(2*Y3/s^2)+(Y2/s^4)),5);
r = input('Enter r value at support 2 r:');
q = input('Enter q value at support 2 q:');
k = (1.5*r^2-2.5*r^3+r^4)*(0.5*q-1.5*q^3+q^4);
echo on
%Edge moment
echo off
beta = vpa(-u*(v*(3-15*r+12*r^2)*(0.5*q-1.5*q^3+q^4)+(1/s.^2)*(1.5*r-
2.5*r^3+r^4)*(12*q^2-9*q)),5);
mxe = vpa(beta*Q*a^2,5)
%for CCCS span2
```

```
syms r q
U = 1.5*r^2-2.5*r^3+r^4;
V = q^2-2*q^3+q^4;
diff(U,2);
(diff(U,2))^2;
y1 = int((diff(U,2))^2,r,0,1);
z1 = int(V^2,q,0,1);
Y1 = y1*z1;
diff(V,2);
(diff(V,2))^2;
y2 = int(U^2,r,0,1);
z2 = int((diff(V,2))^2,q,0,1);
Y2 = y2*z2;
diff(U,1);
diff(V,1);
y3 = int((diff(U,1))^2,r,0,1);
z3 = int((diff(V,1))^2,q,0,1);
Y3 = y3*z3;
y4 = int(U,r,0,1);
z4 = int(V,q,0,1);
Y4 = y4*z4;
u1 = vpa(Y4/(Y1+(2*Y3/s^2)+(Y2/s^4)),5);
r1 = input('Enter r1 value at support 2 r1:');
q1 = input('Enter q1 value at support 2 q1:');
k1 = (1.5*r1^2-2.5*r1^3+r1^4)*(q1^2-2*q1^3+q1^4);
echo on
%Edge moment
echo off
beta1 = vpa(-u1*(v*(3-15*r1+12*r1^2)*(q1^2-2*q1^3+q1^4)+(1/s.^2)*(1.5*r1-
2.5*r1^3+r1^4)*(2-12*q1+12*q1^2)),5);
mxe1a = vpa(beta1*Q*a^2,5)
mxe1b = mxe1a
%for CCSS span3
syms r q
```

```
U = 1.5*r^2-2.5*r^3+r^4;
V = 1.5*q^2-2.5*q^3+q^4;
diff(U,2);
(diff(U,2))^2;
y1 = int((diff(U,2))^2,r,0,1);
z1 = int(V^2,q,0,1);
Y1 = y1*z1;
diff(V,2);
(diff(V,2))^2;
y2 = int(U^2,r,0,1);
z2 = int((diff(V,2))^2,q,0,1);
Y2 = y2*z2;
diff(U,1);
diff(V,1);
y3 = int((diff(U,1))^2,r,0,1);
z3 = int((diff(V,1))^2,q,0,1);
Y3 = y3*z3;
y4 = int(U,r,0,1);
z4 = int(V,q,0,1);
Y4 = y4*z4;
u2 = vpa(Y4/(Y1+(2*Y3/s^2)+(Y2/s^4)),5)
r2 = input('Enter r2 value at support 4 r2:');
q2 = input('Enter q2 value at support 4 q2:');
k2 = (1.5*r2-2.5*r2^3+r2^4)*(1.5*q2^2-2.5*q2^3+q2^4);
echo on
%Edge moment
echo off
beta2 = vpa(-u2*(v*(3-15*r2+12*r2^2)*(1.5*q2^2-2.5*q2^3+q2^4)+(1/s.^2)*(1.5*r2^2-
2.5*r2^3+r2^4)*(3-15*q2+12*q2^2)),5);
mxe2a = vpa(beta2*Q*a^2,5)
echo on
%Fixed end moment at each support(starting from left to right of the plate span)
echo off
f1 = 0
```

```
f2 = mxe-mxe1a
f3 = mxe1b-mxe2a
f4 = 0
%using element stiffness method of analysis
E = input('young modulus E:');
I = input('second moment of inertia I:')
%flexural rigidity (EI) = FR
FR = E*I;
A = [4 2; 2 4];
%element stiffness
K1 = A.*FR;
K2 = K1;
K3 = K1;
%global stiffness K
K = [4 2 0 0 ; 2 8 2 0 ; 0 2 8 2 ; 0 0 2 4];
f = Q*a^2
B = [f1; f2; f3; f4];
F = B.*f
echo on
%F = K*theta, hence, theta = inv(K)*F
%theta = [theta1; theta2; theta3; theta4]
echo off
theta = K\F
%For member forces(md)
%md = [md..; md..]
echo on
%enter the valuues of terms of theta matrix above.
echo off
theta1 = input('theta1:');
theta2 = input('theta2:');
theta3 = input('theta3:');
theta4 = input('theta4:');
thetaa = [theta1; theta2];
thetab = [theta2; theta3];
```

```
thetac = [theta3; theta4];
md12 = K1*thetaa
md23 = K2*thetab
md34 = K3*thetac
%Fixed end or edge moment
fem = [0; mxe; mxe1a; mxe1b; mxe2a; 0];
echo on
%Final Support moment(m)
echo off
%m = fem-md, m12 =
fem12 = [0; mxe];
fem23 = [mxe1a; mxe1b];
fem34 = [mxe2a; 0];
m12 = fem12-md12
m23 = fem23-md23
m34 = fem34-md34
%Average final Support monent(mavs) is mavs... = (m21+m22)/2
echo on
%input the positive values of m matrix above. note: m12 = [ms11;ms21], m23 = [m22;m32]
...
echo off
ms11 = input('Enter monent at support 1 member 1:');
ms21 = input('Enter monent at support 2 member 1:');
ms22 = input('Enter monent at support 2 member 2:');
ms32 = input('Enter monent at support 3 member 2:');
ms33 = input('Enter monent at support 3 member 3:');
ms43 = input('Enter monent at support 4 member 3:');
echo on
%average final support moment
echo off
mavs1 = ms11
mavs2 = (ms21+ms22)*0.5
mavs3 = (ms32+ms33)*0.5
mavs4 = ms43
```

```
echo on
%Span Moment(mspan)
echo off
%mspan = 0.125*Q*a^2-(mst+mst)*o.5
c = 0.125*Q*a^2;
mspan12 = c-((mavs1+mavs2)*0.5)
mspan23 = c-((mavs2+mavs3)*0.5)
mspan34 = c-((mavs3+mavs4)*0.5)
R = 0:0.166666666:1;
bm = [mavs1 -mspan12 mavs2 -mspan23 mavs3 -mspan34 mavs4];
plot(R,bm,'r-')
grid
xlabel('R');
ylabel('bm');
title('BMD for strip 1b-1b 2way Continuous plate')
```